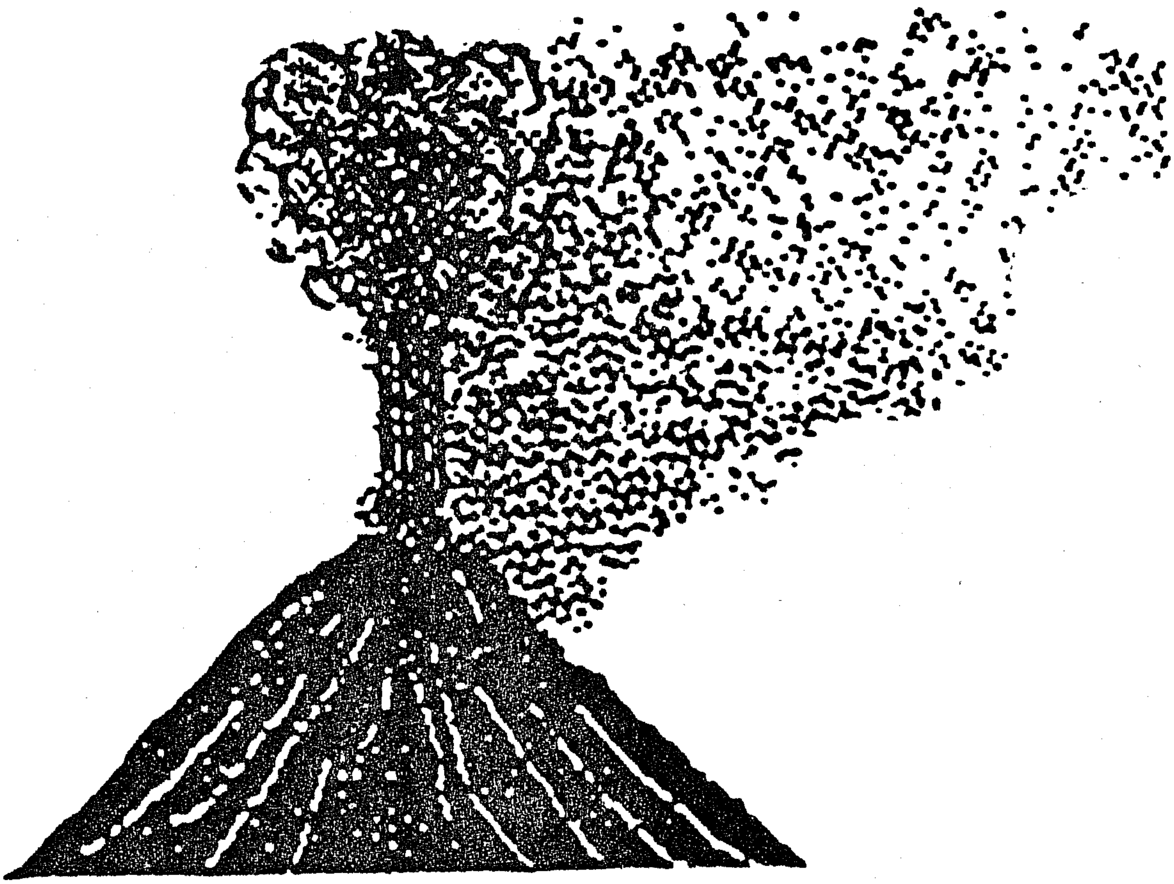
Volcanic Eruption Fallout
TEACHER RESOURCE BOOK

written by
Jeanne Agnew
Professor of Mathematics
Oklahoma State University

graphics by
Jody Jobe
New York City, NY



This book is one part of a complete learning module for the problem "Volcanic Eruption Fallout." The entire learning module consists of three video cassettes entitled "The Problem," "Problem Preparation," and "A Solution"; a student resource book; a teacher resource book; and a microcomputer diskette for use on an Apple computer.

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The materials produced under the AIM project are based on industry-related applied mathematics problems. They have been designed and produced to offer high school teachers a strategy for providing their students an experience in using their reading, writing, and mathematical abilities to solve real problems. In this way the students may realize that these skills are interrelated and that the mastery of them is of vital importance in their future career opportunities.

For more information about this and other AIM modules, write to:

AIM: The Mathematical Association of America
1529 Eighteenth Street, NW
Washington, DC 20036

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Title: Volcanic Eruption Fallout

Source: Dr. William I. Rose
Professor of Geology
Michigan Technological University
Houghton, Michigan

Prerequisites: Current enrollment in Algebra II; a course in some science would be helpful but not necessary

Skills Needed: Arithmetic, ability to understand and manipulate given algebraic formulas, ability to adjust units

Summary: Dr. Rose is a volcanologist. This problem concerns the distribution of fallout from the eruptions at the Augustine Volcano, March 27 and 28, 1986. A mathematical model is given and the students are asked to calculate the predictions from the model and compare them with real data.

Comments: Necessary background information is provided. The students get experience in using mathematical skills to analyze an unfamiliar situation. The problem gives the students a unique experience in working with a mathematical model. They find that a "wrong" answer judging by the model may indeed be "right" and may provide useful information.

Suggested Classroom Uses:

A unit in a mathematics course (such as Algebra II, or Analysis)

Independent study

A project for a math club

Enrichment in Algebra II, or Earth Sciences

Experience in keeping units correct

Experience in manipulating algebraic expressions

Experience in computer simulation

Volcanic Eruption Fallout
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I. Introduction

This book is the Teacher Resource Book for the industry-related problem "Volcanic Eruption Fallout." It is one part of a coordinated package of materials called an AIM Learning Module. The total AIM module consists of: Video I, "The Problem"; Video II, "Problem Preparation"; Video III, "A Solution"; the Student Resource Book; the Teacher Resource Book; and a microcomputer BASIC diskette.

All three videos are conversations with the volcanologist who has actually worked with this problem. The videos were filmed in Alaska at the site of the volcano.

In order to understand the background of this problem the student should have completed or be enrolled in Algebra II. Section II of the Student Resource Book consists of general background information regarding volcanic eruptions and mathematical models. Section III presents the equations and formulas that are relevant to the problem. The needed data is also given in Section III. In order to ensure that the student has grasped the important concepts, Section III provides a series of TOPs (Try Out Problems) through which the student can check how well the ideas have been mastered. Section IV is a statement of the general problem; Section V includes the particular problems.

Section II of the Teacher Resource Book, entitled "Teaching Strategies," describes a variety of situations where an AIM learning module can be used. Section III, "Where to Begin," discusses ways in which the teacher can introduce this particular problem and points out some activities which can help the students get started. Section IV gives answers to the TOPs.

Section V, entitled "Developmental Approach," sets forth the thought processes through which a student might proceed in reaching a solution to the problem. Since one way of formalizing your thinking is to ask yourself a series of questions, the developmental approach is written in the form of questions the students may ask themselves. This section can be used by the teacher as a source of leading questions to guide student discussion of the problem in a class. Or it can be given to the students to help them in thinking. If the problem is to be used as independent study, this section will be especially useful. It is probable that for the student's first experience with an AIM module the developmental approach will be important. After the student has worked through one or two of the AIM modules, the student may have acquired enough problem-solving techniques to make the developmental approach unnecessary.

Section VI, "A Solution in Detail," gives a complete solution to each problem and makes some comments on questions the students may ask. Section VII discusses the computer programs provided and explains ways in which the initial information can be altered to answer questions which might arise. In Section VIII the writing of a report is discussed.

Section IX is a collection of additional questions. Included in this section are "What if" questions that explore the effects of changing the parameters involved in the problem.

II. Teaching Strategies

There are many exciting ways in which the AIM materials can be used in the high school curriculum. You are, of course, free to use your creativity to modify these and to devise others that fit your individual situation.

Whatever method you choose, the goal is two-fold: to have the students experience mathematics in an industrial setting; to raise the student's awareness of careers in mathematics.

A. A Unit in a Mathematics Course

The objective is to involve the student in a discovery-learning approach geared to developing and sharpening the following skills:

1) reasoning and model building
2) real-world problem solving
3) communicating verbally about mathematics
4) writing technical material concisely and accurately
5) making use of resource materials
6) using the computer in problem solving.

Step 1. Preparation. The teacher gives each student a copy of the Student Resource Book. (Permission is granted to copy these for classroom use.) The student reads it and becomes acquainted with the problem and its setting.

Step 2. Video I. The students view Video I. Through this on-site video they meet the volcanologist who discusses the problem and some related material.

Step 3. Getting Started. The students study the problem, become familiar with the terms and consult reference books as needed. They talk about the problem, its setting, the technical terms, the assumptions made, and ways in which they might attack the problem. At some point the students may feel the need for reinforcement and extra direction. They then view Video II, where hints are given by the volcanologist to help them solve the problem.

Step 4. Creating a Solution. The students work as a class or as individuals and discuss their work. At this stage you, as teacher, skillfully nudge the students toward a solution. To help you accomplish this, Section V of this book, entitled "Developmental Approach," provides a succession of questions some of which you might use to stimulate discussion. A complete solution of the problem is provided in Section VI. However, the teacher's role is not to provide a solution, but rather to encourage and tease the students to find their own solution. Students interact with their peers and their instructor and also use the resources of their campus (library, computer center, faculty, etc.). Students come to class ready to report on their mathematical progress. When solutions are obtained, they present their solutions and field questions on their work.

Step 5. The Computation. A solution program is provided on a diskette for use on an Apple II. The program is user-friendly and allows the student to enter a variety of initial data.

Step 6. The Report. Each student writes a technical report on the problem and its solution. The report is discussed in Section VIII and a suggested format is given in Section XI.

Step 7. Video III. The students view Video III, "A Solution." At this time they can compare their solution with that provided on the video.

B. A Unit for Independent Study

The method described in A is an "ideal" way to use the AIM materials in a class-oriented problem-solving situation. The same general method is equally effective when used as individual instruction or independent study for one or more students. In such a case, class discussion gives way to periodic teacher conferences. If the student is short of time or unable to proceed, the teacher can provide the list of questions given in the developmental approach (Section V) to lead the thought processes of the student.

C. Enrichment in a Variety of Courses

Use AIM Videos I, II, and III as a lecture presentation to a class when you wish to stimulate interest in mathematics by demonstrating an application of the material they are studying.

Assign parts of the problem when the class work deals with some skill used in the problem solution. In this case you might show AIM Video I to acquaint the class with the problem setting and then give a brief discussion of the method of solution.

Students can experience computer simulation by using the computer solution with different data. The "What if" problems suggested in Section IX illustrate the use of the computer to answer questions regarding the effects of changing quantities such as the particle diameter.

D. Project for a Math Club

The AIM materials make an exciting series of programs for a math club. The technique described in Section A can easily be adapted to this setting. The fact that math clubs include students at various levels adds interest in the sharing of skills.

E. Developing Career Awareness

All three videos give a firsthand picture of a volcanologist at work. Implicitly all three raise the students' consciousness of the importance of mathematics in their future career choices.

F. Group Presentations

Use the AIM Video as the basis of a presentation at a regional meeting of NCTM.

Use AIM Videos and written materials as a resource for a workshop for your area group of mathematics teachers.

III. Where to Begin

A major hurdle in problem solving is to decide where to begin. The first formal step is to read through the Student Resource Book. The material may seem somewhat forbidding at first reading. Viewing Video I should help allay fears and arouse interest. The students might be encouraged to check their school or public libraries for books on volcanoes. Books on aerosol science would also be helpful for students who wish to explore further the behavior of fallout particles.

In this problem it is vital that the students pay close attention to the appropriate units of measurement for the particular formula or equation they are using. For example, in Section III B of the Student Resource Book, Wilson's equation yields a column height in meters only when the mass eruption rate is measured in kg/s. Also, in Section III D, equation (3) is an equality only if the units on both sides of the equation agree.

Encourage students to work through each example and each TOP. This is essential in order to acquire familiarity with the formulas and ideas that are presented. Suggest that they make a list of important formulas, a glossary of unfamiliar terms, and a list of the symbols being used. A sample glossary, list of formulas, and list of symbols are included in the appendix.

At some point, as the students are working toward understanding the setting, they may wish to view Video II for a few hints and additional emphasis on the pertinent facts.

Once the overall picture is understood, the REAL fun begins. Although all necessary information is provided and emphasis is placed on important ideas, the procedure is not laid out for the students. It is up to them to decide how to make the calculations and to compare the mathematical model with the actual data.

IV. Answers to TOPs

1.

(a) $1 \text{ m} = 100 \text{ cm}$
$1 \text{ m}^3 = (100)^3 \text{ cm}^3 = 10^6 \text{ cm}^3$
$\text{mass} = (2.5 \text{ g/cm}^3)(10^6 \text{ cm}^3) = 2.5 \times 10^6 \text{g}$

(b) $\text{Volume} = (4/3)\pi(\text{radius})^3 = (4/3)\pi(1.9)^3 \text{ cm}^3$
$= 28.73 \text{ cm}^3$

$\text{mass} = 14.4 \text{ g}$

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{14.4 \text{ g}}{28.73 \text{ cm}^3} = 0.50 \text{ g/cm}^3$$

(c) $1 \text{ cm}^3 = 10^3 \text{ mm}^3$

$$\text{density} = 2.5 \text{ g/cm}^3 = \frac{2.5}{10^3} \text{ g/mm}^3$$
$$= 2.5 \times 10^{-3} \text{ g/mm}^3$$

(d) $6.398 \text{ t} = 6.398 \times 10^6 \text{ g}$

$$\text{volume} = \frac{\text{mass}}{\text{density}} = \frac{6.398 \times 10^6 \text{ g}}{2.5 \text{ g/cm}^3}$$
$$= 2.56 \times 10^6 \text{ cm}^3$$

2.

(a) $7.5 \times 10^7 \text{ t/day} = 7.5 \times 10^7 \times 10^6 \text{ g/d}$
$= 7.5 \times 10^{13} \text{ g/d}$

$1 \text{ day} = 24 \times 60 \times 60 \text{ seconds} = 86400 \text{ s}$

$$\text{mass/second} = \frac{7.5 \times 10^{13} \text{ g/d}}{86400 \text{ s/d}}$$
$$= 0.868 \times 10^9 \text{ g/s}$$

$\text{density} = 2.5 \text{ g/cm}^3 = 2.5 \times 10^6 \text{ g/m}^3$

$$\text{eruption rate} = \frac{\text{mass per second}}{\text{density}}$$
$$= \frac{0.868 \times 10^9 \text{ g/s}}{2.5 \times 10^6 \text{ g/m}^3}$$
$$= 347.22 \text{ m}^3\text{/s}$$

(b) This is lowest.

3. $\frac{T}{\eta} = \frac{5 \text{ cm/s}}{1.81 \times 10^{-4} \text{ g/cm-s}} = \frac{5 \times 10^4}{1.81} \times \frac{(\text{cm})(\text{cm})(\text{s})}{(\text{s})(\text{g})}$

$= 2.8 \times 10^4 \text{ cm}^2/\text{g}$

4.

(a) Units are $\text{cm} \times \frac{\text{cm}}{\text{s}} \times \frac{\text{g}}{\text{cm}^3} \times \frac{\text{cm-s}}{\text{g}}$

and units all cancel.

(b) $\text{Re} = \frac{(22 \times 10^{-4} \text{cm})(4 \text{ cm/s})(0.5 \times 10^{-3} \text{ g/cm}^3)}{1.33 \times 10^{-4} \text{ g/cm-s}}$

$= 0.03308$

The flow is streamline.

5. Units on the left hand side are:

$(\text{cm}^3)(\text{g/cm}^3)(\text{cm/s}^2) = \text{g-cm/s}^2$

Units on the right hand side are:

$(\text{g/cm}^3)(\text{cm}^2)(\text{cm}^2/\text{s}^2) = \text{g-cm/s}^2$

6.

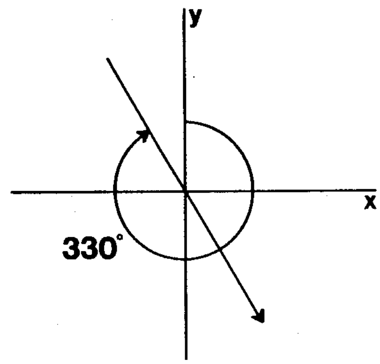
(a) 150 knots = 150 nautical miles/hr

$= 150 \times 1.852$ km/hr

$= \frac{277.8 \text{ km}}{\text{hr}} \times \frac{\text{hr}}{3600 \text{ s}}$

$= 0.0772$ km/s

(b) It would be cold since it is blowing from the northwest.



V. Developmental Approach

A. Preliminary Problems

1. What is Wilson's equation?
What are the units for Q?
What are the units for H?
From what point is H measured?
What is the altitude of the mouth of the volcano?
What must I remember about the horizontal scale when I plot this point on the graph?

2. From what altitudes can particles begin to fall?
When drawing the wind profile, do units matter?
What units do I need in my calculations?
What units are most efficient?

B. The Problem

1. How can I find a formula for T?
   Would writing the given formulas be a good idea?
   Do any letters in equations (1), (2), and (3) have formulas representing them? I should write these down. If I solve (3) for T do I get a simple expression?
   Is this really a formula for T?
   Some symbols represent constants and some change with altitude. What about σ? g? η? d? ρ? C? Re?
   Where does C come from? Where does Re come from?
   Let's get rid of C.
   It looks as if I might easily go around in circles.

   Suppose I form a plan.
   I should reread Section III D. Which equation expresses the basic relationships between the forces operating on the particles?
   If (3) is the basic equation, I should substitute from (1) and (2) into (3).
   Now what have I found? Is this a formula for T or is it a new equation?
   It is an equation. Can I solve the equation for T?
   This gives me two values for T. Are both possible?

2. Now I can calculate T in each atmospheric layer.
   How long does it take a particle to fall through one layer? What is the distance? What is the vertical speed?
   If speed is constant, distance = speed x time.
   What carries the particles horizontally?
   Is the horizontal speed constant in each layer?
   How long is the particle in a given layer?
   How far does it go horizontally?
   This graph should be easy to draw once I decide on a scale for each axis. Which is larger, the horizontal or the vertical distance?

3. How long does it take particles in the 8-9 km layer to reach the ground?
   How far have they gone horizontally?
   Could particles from the 8-9 km layer reach Anchorage in 22 hours?
   From which layer do particles reach the ground in 22 hrs? I can work upwards through 0-1, 1-2, 2-3, etc. until I find out.
   What are the corresponding horizontal distances?

Do any particles begin in the 0-1 layer?
Where must particles begin if they reach the ground at Anchorage?

4. What does the mathematical model say about time of fall and horizontal distance?
Do the model and the real data agree?
What can I say about the discrepancy?
What assumptions are made in using the model?
How can I check whether these are justified?

C. Computer Problem

Writing the computer problem to represent the formulas is not too difficult. Could I have my program make calculations that would help me in step 3?

VI. A Solution in Detail

The main difficulties students are likely to have with this problem are identifying the correct formulas, and keeping the units correct. It will help if they make a careful list of appropriate formulas and also a list of notation. They should carry the units in their calculations. They should work the TOPs in order to become familiar with the formulas.

A. Preliminary Problems

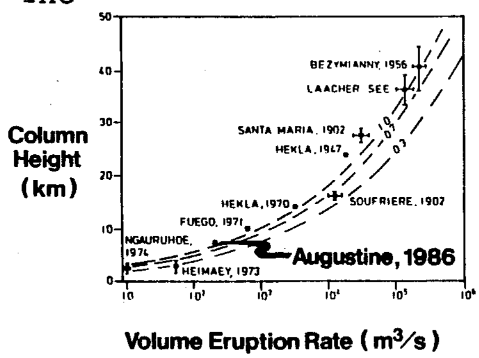
1. (a) Wilson's equation $H = 236.6(Q)^{1/4}$ gives H in meters when Q is measured in kg/s. The data gives Q in metric tons per day. The first step is thus to convert t/d to kg/s. We have

$$7.5 \times 10^7 \text{ t/d} = \frac{7.5 \times 10^{10} \text{ kg}}{24 \times 60 \times 60 \text{ s}}$$
$$= 86.806 \times 10^4 \text{ kg/s}.$$

$$(Q)^{1/4} = 3.0524 \times 10 = 30.524$$

$$H = 236.6 \times 30.524 = 7.222 \text{ km}.$$

Figure 1



(b) The height is 7.222 km above the crater of the volcano. The mass eruption rate

$$= \frac{7.5 \times 10^7}{24 \times 3600} \text{ t/s}$$
$$= \frac{7.5 \times 10^{10}}{24 \times 3600} = 8.6806 \times 10^5 \text{ kg/s}$$
$$= 8.6806 \times 10^8 \text{ g/s}$$

Figure 2

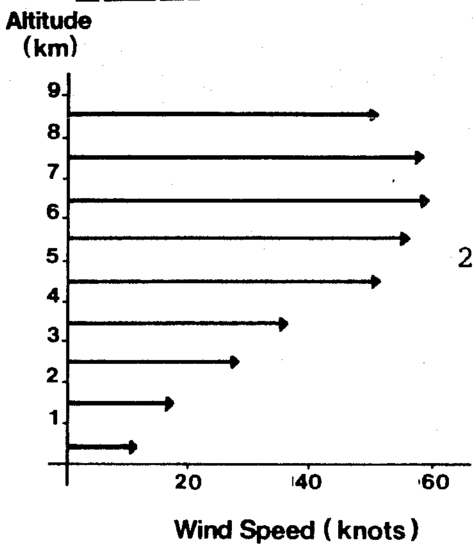
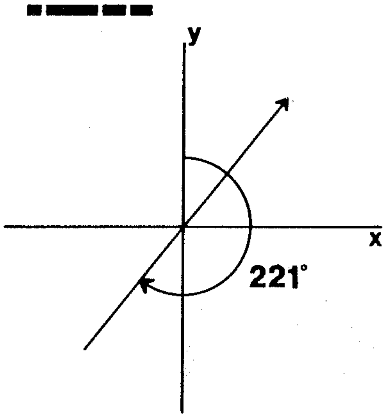


Figure 3



Wind Direction

The volume eruption rate is

$$\frac{8.6806 \times 10^8 \text{ g/s}}{2.5 \text{ g/cm}^3} = 3.4722 \times 10^8 \text{ cm}^3\text{/s}$$

$$= 3.4722 \times 10^2 \text{ m}^3\text{/s}$$

2. (a) The wind profile should show nine arrows. The length of the arrows should correspond to the wind speed. See Figure 2.

(b) The prevailing direction is found by adding the azimuth as given in Column 2, Table 3, and dividing by 9. The prevailing direction is 221°. Students should notice that the wind was blowing consistently in approximately this direction. Such a situation would not always hold.

B. The Problem

1. We have three formulas:

(1) $\quad \text{Re} = \dfrac{d\ T\ \rho}{\eta}$

(2) $\quad C = 24/\text{Re}$

(3) $\quad V\ \sigma\ g = \dfrac{1}{2}\ \rho\ C\ A\ T^2$

Since both V and A involve π and r, the students might wish to begin by simplifying (3):

(4) $\quad (4/3)\pi r^3\ \sigma\ g = (1/2)\rho\ C\pi r^2\ T^2$

Multiply both sides by $\dfrac{6}{\pi r^2}$:

(5) $\quad 8r\ \sigma\ g = 3\rho\ C\ T^2$

From (1) and (2) $\quad C = \dfrac{24\ \eta}{d\ T\ \rho}$

Substitute for C in (5)

(6) $\quad 8r\ \sigma\ g = \dfrac{3\ \rho\ 24\ \eta\ T^2}{d\ T\ \rho}$

The students should question whether T can be divided out of the right hand side. The case T = 0 is not pertinent to this problem, though it may in fact be the case that very fine particles are suspended in the air and do not fall to the ground. After T and ρ are cancelled, the formula becomes

$$8r \sigma g = \frac{72 \eta T}{d}$$

The students should now see the advantage of replacing 8r by 4d so that the following formula for T is obtained:

(7) $$T = \frac{d^2 \sigma g}{18 \eta}$$

Comment: There are many ways in which formula (7) can be reached. Some students may try to replace all the known quantities by their values. With this approach they will soon find that certain quantities change from layer to layer. It is best to encourage them to work in the general case.

An alternate route is to use (3) to solve for T:

$$T^2 = \frac{2 V \sigma g}{\rho C A}$$

$$T^2 = \frac{2(4/3)\pi r^3 \sigma g}{\rho C \pi r^2}$$

This is not really a formula for T since C involves Re which in turn involves T. If they now eliminate C and simplify, they obtain

$$T^2 = \frac{8 r \sigma g \text{ Re}}{3 \times 24 \rho}$$

Now use (1) to substitute for Re. The remaining steps are reasonably clear:

$$T^2 = \frac{8 r \sigma g d T \rho}{72 \eta \rho}$$

$$T^2 - \frac{r \sigma g d T}{9 \eta} = 0$$

$$T(T - \frac{\sigma r g d}{9\eta}) = 0$$

$$T = 0, \text{ or } T = \frac{\sigma r g d}{9\eta}$$

Since T = 0 means the particles do not fall, for our problem we wish the other choice. We write r as d/2 and obtain:

(7) $$T = \frac{d^2 \sigma g}{18\eta}$$

2. (a) Now that formula (7) is available, the calculation of terminal velocity is straightforward using the values of d, σ, g, and η given in Section III G. The calculated values of T are listed in Table 4. The results obtained by students may vary slightly depending on the rounding techniques used.

(b) The time of vertical fall in each layer is found by dividing the distance (1 km) by the terminal velocity for that layer. For example in the layer from 0 to 1 km, T is 3.97 cm/s so that the time is

$$\frac{10^5 \text{ cm}}{3.97 \text{ cm/s}} = \frac{100000 \text{ cm}}{3.97 \text{ cm/s} \times 3600 \text{ s/hr}}$$

$$= 7.00 \text{ hours.}$$

The times are listed in Column 3 of Table 4, and the total time is the total of Column 3.

(c) In order to calculate the total distance travelled horizontally in each layer we need

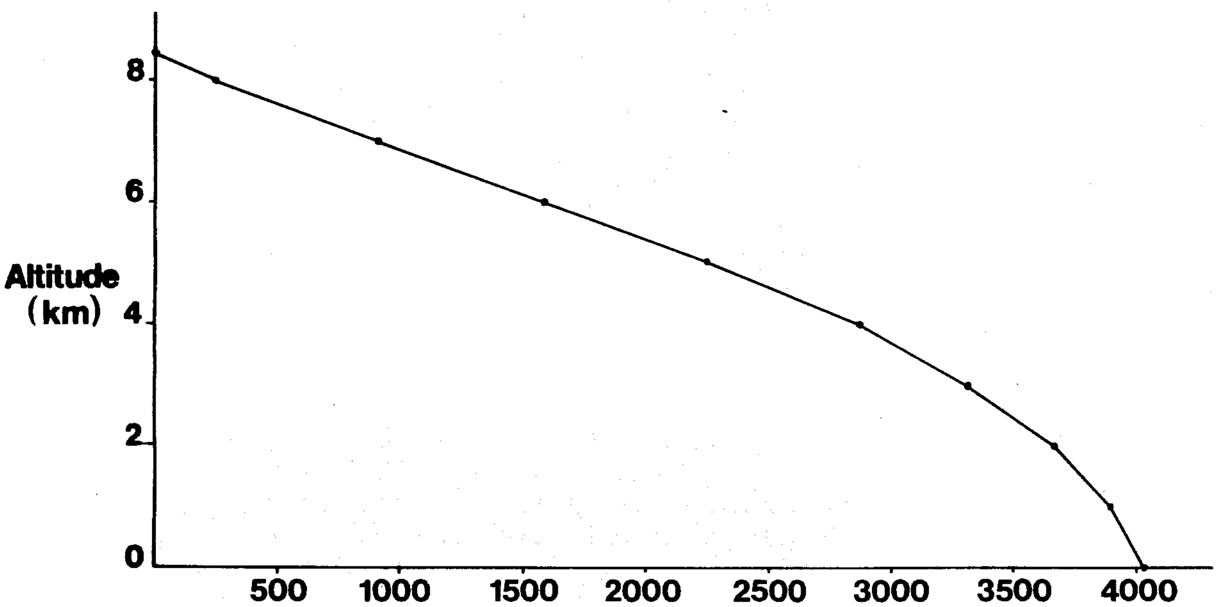
(time of fall through layer)

x (wind speed in that layer).

In order to get the distance in kilometers it is necessary to convert knots to km/hr. Recall that 1 knot is 1 nautical mile per hour, that is, 1.852 km/hr. The wind speed in km/hr is listed in Column 4 of Table 4, and the horizontal distance is found in Column 5. The total horizontal distance is the total of Column 5.

(d) This graph is a set of line segments. These segments differ very little in slope until the lower layers are reached. See Figure 4.

Figure 4



Fallout Trajectory Beginning at 8.42 km Above Sea Level

3. (a) Particles falling from the 8-9 km layer need 59.15 hours to reach the ground. They will not reach the ground in 22 hours anywhere. When they do reach the ground the model says they are 4373 km northeast of the volcano, well beyond Anchorage.

(b) Column 3 in Table 4 gives the answer to this question. A particle from 3 km will require 20.66 hours to reach the ground. Therefore to reach the ground in 22 hours the particle should have started above 3 km. In this case it will have travelled more than 717 km.

(c) Particles that fall from 2 km above sea level have already moved 365 km northeast and have passed Anchorage. In order to reach Anchorage a particle would need to start from the level of the crater of the volcano.

4. Since the actual situation involved a measurable amount of fallout in Anchorage in 22 hours, the model does not agree with the real data. Even those particles that rose no more than 3 km took 20 hours to reach the ground and were beyond Anchorage when they did so.

The students should first check the assumptions made in using equation (3). Is the Reynolds number substantially less than 1? The atmospheric data needed to calculate Re in each atmospheric layer is available in Table 2, Section III G. This is a straightforward calculation by computer or hand calculator. The student might guess that it is not really necessary to calculate all of them. The Reynolds numbers vary from 0.00304 to 0.0065.

The other assumption that was required was that the particles were spherical in shape and fell independently. This will be a good topic for student discussion especially if some students become sufficiently interested to do outside reading or to consult a science teacher. Dr. Rose will discuss it in the third segment of the accompanying video. He will suggest that the fine ash particles are attracted to each other by electro-magnetic forces and fall as adhesive groups that look somewhat like snowflakes but separate when they reach the ground.

The particles, if examined closely under a microscope, are seen not be exactly spherical. The mathematical model becomes much more complicated if an effort is made to adjust for a nonspherical shape. Since the spherical shape has the least air resistance, making the assumption that the shape is spherical increases the value of T. In our calculations, the time of fall and the distance travelled were larger than the actual data. Thus for our calculations using the spherical shape gave the best likelihood of fit.

Assumptions were also involved in considering the atmosphere as consisting of 9 layers. Because there is so little change from one layer to the next, it is reasonable to believe that no significant change would result if more layers were to be considered.

TABLE 4

PATTERN OF FALLOUT

Altitude km	Terminal Velocity cm/s	Time of Fall hrs	Wind Speed km/hr	Horizontal Distance km
8-9	4.45	6.24	93.53	583.7
7-8	4.42	6.28	107.42	674.9
6-7	4.39	6.32	108.34	685.2
5-6	4.31	6.45	100.93	651.2
4-5	4.25	6.54	94.45	617.3
3-4	4.17	6.66	66.67	444.2
2-3	4.09	6.79	51.86	352.0
1-2	4.04	6.87	32.41	222.8
0-1	3.97	7.00	20.37	142.6
Totals:		59.15		4373.9

Comment: A point which students might discuss is whether or not the atmospheric layer from 8-9 km should be included in the calculation of total time. Some may wish to stop with the 7-8 km layer, and some may wish to include only a portion of the highest layer (about 2/5). They should be encouraged to try all three and compare their numbers. In the light of the accuracy of the information, does it make any difference in this problem?

Students may come up with slightly different answers depending on the methods of rounding used. In calculating Table 4, the numbers are rounded when printed, but are not rounded during calculations.

VII. The Computer Programs

There are two computer programs included in the diskette for this AIM module. They are written in BASIC for the Apple II computer. To run them, boot up the diskette and type "RUN" and the name of the program you wish to use.

The first program, FALLOUT TABLES, provides information regarding fallout rate and distribution. The user is asked to enter numerical values for the diameter and the density of the ash particles. The user then has four options: A) Print the fallout tables; B) Print the Reynolds number and drag coefficient for each atmospheric layer; C) Enter new data; D) Quit.

Option A allows the user to view two tables. The first shows the pattern of fallout through each of the nine layers, from 0-1 km, up to 8-9 km. The table shows the terminal velocity in cm/s, the time of fall in hours, the wind speed in km/hr, and the horizontal distance travelled in km. The second table shows the cumulative pattern of fallout, including the total time of fall from each layer to sea level, and the total horizontal distance travelled (km) by particles from each layer before reaching the ground.

If the user chooses option B, the program uses the diameter and density already input by the user, and calculates the Reynolds number and drag coefficient at each of the nine atmospheric layers.

Option C allows the user to change the input data without exiting the program, and then run option A or B for the desired information.

The second program, titled COLUMN HEIGHT, asks the user to input the mass eruption rate in metric tons per day. The program first calculates the column height from the crater of the volcano, corresponding to this eruption rate. It then produces a graphic representation of an eruption of the correct height.

VIII. The Written Report

The ability to communicate clearly, both orally and in writing is an important skill in any area. Report writing is a regular part of the job of a person working in industry. One of the skills gained from the use of the AIM problems should be an increased ability to read with understanding and to write clearly and accurately.

In this AIM problem the student is playing the role of a member of a team led by a volcanologist. This role includes writing a report stating the work requested, the work done, and the results found. Encourage students to be brief, precise, and clear. A suggested format is given in the appendix.

The following suggestions may be helpful to you in making report writing a meaningful experience.

1. It is important that the student know from the beginning that a written report is expected and what the report should cover. For this AIM module the report might concern comparing real data with data from a mathematical model of the distribution of fallout from the Augustine volcano.
2. Writing the report should begin with the Body. (See Report Format, Appendix C.) Have the student write the introduction as soon as the student realizes what is to be calculated. The introduction would include only three or four sentences describing the problem. For example: Since volcanic eruptions can be very dangerous, it is important to try to predict the results of eruptions. Mathematical models are often used to predict such events. We are asked to compare predictions from a mathematical model of the distribution of fallout from Augustine with the actual data observed at Anchorage.
3. After the students have studied their resource books and watched the first video they can begin the Discussion. What assumptions are made? What notation is used?
4. Have the student hand in the Introduction and the first part of the Discussion at this time. In this way the report is actually begun and you can be sure the student understands what is given and what is to be done.

5. As the student works on the problem things needed in the Discussion section can be listed. The formulas needed should be included. Be sure there is a sentence introducing each formula telling in words what it gives. For example: "The drag coefficient is defined in terms of the Reynolds number by the formula: ..."

6. The details of the algebra should be included in the Appendix. Conclusions and Recommendations in this case would be the differences between the predictions from the model and the real data. Some discussion of possible reasons for these differences should be included.

7. The opening material can now be added. The Glossary can be fairly short. There will also be a List of Symbols. The Abstract should include a statement of the problem and one or two sentences on the solution.

8. The References and Acknowledgements can be listed as the work proceeds so that they will be available at report time. References should include articles on volcanoes that were read, and possibly a physical science text.

Although students are rarely delighted at the prospect of writing a report, they are very proud when they see what they have accomplished! Compliment them if at all possible. Suggest that they keep it for their portfolio for use when applying for scholarships or jobs.

IX. Additional Questions

A. Think About Wilson's Equation

1. Under suitable conditions the column height can be measured and the mass eruption rate calculated using Wilson's equation. Find the mass eruption rate
   (a) if the observed height is 4 km
   (b) if the observed height is 11 km.

2. Assume the average density of the erupted material is 2.5 g/cm^3. Derive a formula for converting mass eruption rate Q (t/d) to volume eruption rate Ve (m^3/s). Check your formula by using it to answer TOP 2.

3. The graph of the equation found in question 2 is a straight line. What is the slope of this line? How does the slope change if the density is increased?

4. Wilson's equation was found by fitting a curve to a figure similar to Figure 6 (Student Resource Book). Table 1 (Student Resource Book) is different in origin. Assume that the ejected material in all cases had density 2.5 g/cm^3. Use Wilson's equation to calculate the column height from the eruption rate as given in Table 1.

5. The eruption at Mount St. Helens, May 18, 1980 was observed to reach an altitude of 25 km. The crater was about 3 km prior to the eruption and a little more than 1 km was blasted away in the initial eruption. Assume a column height of 23 km, and an average density of 2.5 g/cm^3.
   a. Find the mass eruption rate in t/d.
   b. Find the volume eruption rate in m^3/s.
   c. Locate the St. Helen's eruption on Figure 6 (Student Resource Book).

B. Think About T

1. Assume the viscosity of the atmosphere is the average of the values given in Table 2 (Student Resource Book) for the region 0-4 km. Calculate T for this region.

2. For the region 0-4 km, what value of T would be needed for the fallout to reach the ground in 22 hr?

3. What diameter is needed to give this value of T, if $\sigma = 2.5$ g/cm^3?

4. Find the Reynolds number for the value of T found in question 2, assuming the diameter found in question 3. Are the assumptions of the model satisfied?

5. What horizontal distance would be travelled by particles falling from 4 km for the T in question 2?

6. The following information was obtained during the eruption at Mount St. Helens, May 18, 1980.

Altitude km	Atmospheric Viscosity g/cm-s	Wind Speed m/s
18-21	1.45×10^{-4}	1.5
15-18	1.44×10^{-4}	3
12-15	1.44×10^{-4}	7
9-12	1.42×10^{-4}	33
6- 9	1.52×10^{-4}	17
3- 6	1.62×10^{-4}	11
0- 3	1.71×10^{-4}	2.5

The eruption occurred at 9:00. The ash fall started in Spokane, WA, at 15:43 and at Missoula, MT, at 20:00. The size of the ash varied from 24 μm to 27 μm. The density of the fallout particles was 2.5 g/cm^3. Spokane is 416 km east of St. Helens and has an elevation of 573 m. Missoula is 621 km east of St. Helens and has an elevation of 976 m. The wind direction would place both Spokane and Missoula in a position to receive ash fallout.

What does the mathematical model predict in this case? Does it agree with the observed data?

C. What If...

1. What if the particles did not fall at all? How long would it take particles in the 8-9 km layer to be directly over Anchorage?

2. If the particles do not fall at all, how long will it take particles in the 3-4 km layer to be directly over Anchorage?

3. Use your computer program, or the program on the diskette, to make a table showing how increasing the diameter of the particles affects the time of fall and the horizontal distance. Assume $\sigma = 2.5\ g/cm^3$ and use values of d from 0.003 cm to 0.01 cm, increasing at intervals of 0.001 cm.

4. How do the Reynolds numbers change when d changes in question 3? Can the model be used for values of d considered in question 3?

5. Assuming other factors remain the same, for what value of d does the Reynolds number become greater than 1?

6. Suppose d = 0.0022 cm and σ changes. How does Re change? How do the total time of fall and the total horizontal distance change? Make a table to illustrate your answer.

7. Why does a change in diameter affect the values of T more than a change in density?

8. What if the wind conditions were different? Use your computer program, or the program on the diskette, to find a height at which the fallout reaches the ground in 22 hr. Find a set of wind conditions that would make this fallout reach the ground within 10 km of Anchorage.

9. What if five spherical particles fell together?
   (a) What would be the diameter of a sphere equal in volume to the five spheres falling together?
   (b) Assuming the same density, will the pattern of fall of such a conglomeration fit the data?

10. Examine what would happen if two spherical particles fell together.

D. Answers

A. THINK ABOUT WILSON'S EQUATION

A1. $H = 236.6\ Q^{1/4}$ so $Q = \left(\frac{H}{236.6}\right)^4$

(a) $Q = 7 \times 10^6$ t/d

(b) $Q = 4 \times 10^8$ t/d

A2. $$Q\ (t/d) = Q \times \frac{10^6\ g/d}{(3600 \times 24\ s/d) \times 2.5\ (g/cm^3)}$$
$$= Q \times 4.63\ cm^3/s$$
$$= Q \times 4.63 \times 10^{-6}\ m^3/s$$

Ve $m^3/s = 4.63 \times 10^{-6}$ Q, where Q is in t/d.

In TOP 2, $Ve = 4.63 \times 10^{-6} \times 7.5 \times 10^7 = 347.22$ m^3/s.

A3. The slope is 4.63×10^{-6}. The slope decreases as density increases.

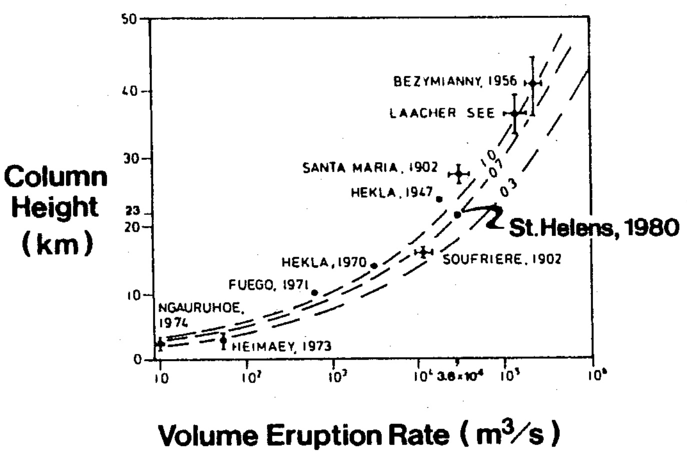
A4.

Eruption Rate		Column Height (km)		
Volume Ve (m^3/s)	Mass Q (kg/s)	Table 1		Wilson's Equation
2×10^4	5×10^7	22(o)		20
2×10^4	5×10^7	24(o)	21(c)	20
3×10^4	7.5×10^7	23(o)		22
2×10^5	5×10^8	45(o)	42(c)	35
4×10^4	1×10^8	34(c)	29(o)	24
6×10^3	1.5×10^7	16(o)	15(c)	15
2×10^3	5×10^6	10(o)	10(c)	11
1×10^6	2.5×10^9	>50(c)		53

A5. Column height = 23 km.

a. Q = 89,300,190 kg/s
$= 7.7 \times 10^9$ t/d.

b. Volume eruption rate $= 7.7 \times 10^9 \times 4.63 \times 10^{-6}$
$= 3.6 \times 10^4\ m^3/s$.



B. THINK ABOUT T

B1. η (average) = 1.62×10^{-4}

T (average) = 4.07 cm/s

B2. Needed value of T is $\frac{4000 \text{ cm}}{22 \text{ hr}}$ = 5.05 cm/s

B3. $$T = \frac{d^2\ (\text{cm})^2\ 2.5\ (\text{g/cm}^3)\ 980\ (\text{cm/s}^2)}{18 \times 1.62 \times 10^{-4}\ \text{g/cm-s}}$$

d = 0.00245 cm

B4. Use ρ (average) = 1.04×10^{-4}

Re = 0.0065. This is within the range for the model.

B5. Horizontal distance travelled is 366.39 + 285.23 + 178.26 + 112.04 = 942.22 km.

B6. Using d = 24 μm

Altitude km	Terminal Velocity cm/s	Time of Fall hrs	Wind Speed km/hr	Horizontal Distance km	Distance and Time from Given Altitude to Sea Level km	hrs
18-21	5.41	15.4	5.4	82.8	4259.3	112.7
15-18	5.44	15.3	10.7	164.4	4176.5	97.3
12-15	5.44	15.3	25.2	385.5	4012.1	82.0
9-12	5.52	15.1	118.7	1791.8	3626.6	66.7
6- 9	5.16	16.2	61.1	987.4	1834.8	51.6
3- 6	4.84	17.2	39.6	682.5	847.4	35.4
1- 3	4.58	12.1	9.1	109.9	164.9	18.2
0- 1	4.58	6.1	9.1	55.0	55.0	6.1

The ash began to fall in Spokane after 6.7 hrs. According to the model no ash could fall in less than 18 hours. Ash from 3-6 km in altitude would travel beyond Spokane before reaching the ground.

Ash fell in Missoula 11 hours after the eruption, sooner than predicted by the model. Ash from layer 3-6 could travel 621 km and fall to 1 km altitude in 25 hours.

Using d = 27 μm

Altitude km	Terminal Velocity cm/s	Time of Fall hrs	Wind Speed km/hr	Horizontal Distance km	Distance and Time from Given Altitude to Sea Level km	hrs
18-21	6.84	12.2	5.4	65.4	3365.3	89.0
15-18	6.89	12.1	10.7	129.9	3299.9	76.9
12-15	6.89	12.1	25.2	304.6	3170.0	64.8
9-12	6.99	11.9	118.7	1415.7	2865.4	52.7
6- 9	6.53	12.8	61.1	780.2	1449.7	40.7
3- 6	6.13	13.6	39.6	539.2	669.5	28.0
1- 3	5.80	9.6	9.1	86.9	130.3	14.4
0- 1	5.80	4.8	9.1	43.4	43.4	4.8

According to the model ash could have reached the ground at Spokane and Missoula if it had begun in the 3-6 km layer. However the time required to do so is longer than the observed time.

C. WHAT IF ...

C1. Time is 280/93.5 = 2.995 hours.

C2. Time is 280/66.67 = 4.200 hours.

C3.

d	Time of Fall	Horizontal Distance	Re
0.003	31.83	2352.2	0.008 - 0.02
0.004	17.9	1323.1	0.02 - 0.04
0.005	11.45	846.9	0.04 - 0.08
0.006	7.95	587.9	0.06 - 0.1
0.007	5.84	432.2	0.1 - 0.2
0.008	4.47	330.6	0.1 - 0.3
0.009	3.53	261.2	0.2 - 0.4
0.01	2.86	211.7	0.3 - 0.6

C4. The Reynolds numbers increase. The model can still be applied.

C5. If d = 0.012, Re = 1.05 for altitude 0-1 km
If d = 0.013, Re $\geqslant$ 1 for altitudes from 0 to 4 km
If d = 0.014, Re $\geqslant$ 1 for altitudes from 0 to 7 km
If d = 0.016, Re $\geqslant$ 1 for altitudes from 0 to 9 km.

C6. Let d = 0.0022 cm

Density	Re	Time of Fall	Horizontal Distance
3.0	0.004-0.008	49.30	3644.8
2.7	0.003-0.007	54.78	4049.9
2.5	0.003-0.006	59.16	4373.9
2.3	0.003-0.006	64.30	4754.0
2.0	0.002-0.005	73.95	5467.1
1.5	0.002-0.004	98.60	7289.8
1.00	0.001-0.003	147.90	10934.6

C7. In the formula for T the diameter occurs to the second power and density only to the first power.

C8. The height is 3.2 km. The horizontal distance travelled is 806.22. To reduce the horizontal distance to 280 km, wind speeds must be reduced to 280/806.22 = 0.35 times their recorded value. Wind speeds are:

0-1 km, 7.13 km/hr
1-2 km, 11.34 km/hr
2-3 km, 18.15 km/hr
3-4 km, 23.34 km/hr

Students may wish to experiment with different amounts of reduction. For example, let a, b, c, d be the wind speeds in layers 0-1, 1-2, 2-3, 3-4 respectively. Then

$$7.00\,a + 6.87\,b + 6.79\,c + 1.332\,d = 280.$$

(1.332 = 0.2 x 6.66, the estimated time of fall in the 3-4 layer.) The numbers a, b, c, d can be chosen in many ways, for example, a = 5, b = 15, c = 18, d = 20.

Probably the simplest approach is to assume the wind speed is the same in the first four layers. Then the required speed is 280/22 = 12.7 km/hr.

C9. For the volumes to be the same, let R be the new radius, and r the radius of each of the five small spheres. Then

$$(4/3)\,\pi R^3 = 5\,(4/3)\,\pi r^3$$
$$R = \sqrt[3]{5}\,r$$
$$R = 1.71\,r = 0.00376 \text{ cm.}$$

Total time of fall is 5.09 hours, distance horizontally is 376.4 km.

C10. Using the same argument as in C9, $R = \sqrt[3]{2}\, r$ so that R = 0.0028 cm.

Total time is 9.13 hours, horizontal distance is 675.2 km.

X. References

This module, like all that deal with real events in our universe, involves some technical terms and some formulas. The Student Resource Book includes all necessary background material, the formulas needed and their meanings, and an explanation of the technical terms. References dealing with the technical aspects of the problem are difficult reading for us as teachers and would probably fail to motivate the students.

On the other hand, we encourage background reading that emphasizes the setting of the problem. This problem gives an introduction to Alaska, the state with which we are probably the least familiar, and an opportunity to encourage the students to read more about its climate, its history, and its potential contributions to our national progress. Students should also be encouraged to read in the general area of earth sciences, particularly in relation to volcanic activity.

Sources for such general information include, of course, encyclopedias, and high school texts in geography and science. Examples of such texts are:

Foster, Robert J., Geology, Columbus, Ohio: Merrill Physical Science Series, 1971.

Hamblin, Kenneth W., The Earth's Dynamic Systems, Minneapolis, MN: Benges Publishing Company, 1975.

Of particular interest relating to volcanoes are the following books:

Coleman, S. N., Volcanoes New and Old, New York: The John Day Company, 1946.

Decker, Robert W. and Barbara, Volcanoes, San Francisco: W. H. Freeman and Company, 1980.

Hebert, Don, and Fluvia Bardassi, Kilauea: Case History of a Volcano, New York: Harper & Row, 1968.

Planet Earth Volcano, Alexandria, VA: Time-Life, 1982.

Powers of Nature, Washington, DC: National Geographic Society, 1978.

Simons, Barbara B., Volcanoes: Mountains of Fire, Milwaukee, WI: Raintree Editions, 1976.

The National Geographic Index lists many articles of interest relating to volcanic activity. Three of these are listed below:

Finley, Rowe, "Eruption of Mount St. Helens," National Geographic, Vol. 159, No. 1, January 1981, pp. 3-65.

Findley, Rowe, "Mount St. Helens Aftermath," National Geographic, Vol. 160, No. 6, December 1981, pp. 713-733.

Tilling, Robert I., "Volcanic Cloud May Alter Earth's Climate," National Geographic, Vol. 162, No. 5, November, 1982, pp. 672-675.

Also of general interest are the PBS TV programs Planet Earth, and the accompanying handbook. The National Geographic TV Special "In the Shadow of Vesuvius" gives a somewhat different perspective. For information on this write National Geographic, WQED-TV, 4801 Fifth Avenue, Pittsburgh, PA, 14213.

XI. Appendix

A. Glossary

Azimuth of a direction - the measure of the angle between the direction and the north direction, measured clockwise from the north.

Density of a substance - its mass per unit volume.

Drag coefficient - a dimensionless measure of atmospheric resistance.

Fallout - the debris that falls to the Earth from a volcanic eruption.

Lava - molten rock ejected from a volcano.

Magma - molten rock under the Earth's surface.

Mathematical model - a set of formulas and equations designed to describe and predict some type of behavior.

Plate tectonics - theory of global-scale dynamics involving the movement of rigid plates of the Earth's crust.

Pyroclastics - solid materials ejected during a volcanic eruption.

Reynolds number - a number that characterizes the flow occurring around a particle which is moving through a fluid such as air.

Terminal velocity - the constant velocity reached by a falling particle when the forces of gravity and atmospheric resistance are balanced.

Viscosity - internal friction due to molecular cohesion in fluids; the property of a fluid that offers resistance to flow. Also, the measure of this property.

B. Symbols, Abbreviations, Formulas, and Equations

LIST OF SYMBOLS

d	diameter of ash particle
σ	density of ash particle
V	volume of spherical particle
A	cross-sectional area of spherical particle
ρ	density of the atmosphere
η	viscosity of the atmosphere
Re	Reynolds number
C	drag coefficient
g	acceleration due to gravity
H	column height of eruption
Q	mass eruption rate
T	terminal (settling) velocity

ABBREVIATIONS FOR UNITS

km	kilometer
nm	nautical mile = 1.852 km
m	meter
cm	centimeter
μm	micrometer = 10^{-4} cm
g	gram
kg	kilogram
t	metric ton = 10^{6} g
d	day
hr	hour
s	second

FORMULAS AND EQUATIONS

1 knot = 1 nautical mile per hour

Volume of a sphere = $(4/3)\ \pi\ (\text{radius})^3$

Cross-sectional area of a sphere = $\pi\ (\text{radius})^2$

Wilson's equation: $H\ m = 236.6\ (Q\ kg/s)^{1/4}$

Reynolds number: $Re = \frac{d\ T\ \rho}{\eta}$

Drag coefficient: $C = 24/Re$, if $Re < 1$

$V\ \sigma\ g = (1/2)\ \rho\ C\ A\ T^2$

C. Report Format

OPENING MATERIAL

Title Page	
Table of Contents	Section headings and page numbers.
Glossary	This should include only terms special to this problem.
List of Symbols	
Abstract	A brief statement of what the student was asked to do and the answer. (No more than four or five lines.)

BODY

Introduction	Statement of what was assigned and why it was needed.
Discussion	The simplifying assumptions. An outline of the formulas given and any formulas that were derived, and an outline of the procedure.
Results	A more detailed statement of the results that are given in the abstract.
Conclusions and Recommendations	If any value judgment can be given, it should be included here.

APPENDIX

References	Any books or articles consulted.
Acknowledgements	Any people who helped.
Computation	A flow chart or program if needed. Detailed results or calculations.

D. Program Listings

```
100  REM      FALLOUT TABLES
110  REM
120  REM    WRITTEN BY PAM CEMEN
130  REM       MARCH 1987
140  REM
150  REM  VARIABLES DECLARATION
160  REM
170  REM  TM(I) = TERMINAL VELOCITY T(I) = TM(I) ROUNDED
180  REM  H = TIME OF FALL HR = H ROUNDED
190  REM  WS(I) = WINDSPEED IN KNOTS W= WS(I) IN KM/HR
200  REM  HD = HORIZONTAL DISTANCE
210  REM  TH(I) = TOTAL HRS OF FALL TD(I) = TOTAL DISTANCE
220  REM  R = REYNOLDS NO. RE = R ROUNDED
230  REM  DC = DRAG COEFFICIENT
240  DIM RHO(10): DIM ETA(10)
250  DIM T(10): DIM WS(10)
260  DIM TM(10):G = 980
270  DIM TH(10): DIM TD(10)
280  HOME : VTAB (8)
290  PRINT  TAB( 7)"APPLICATIONS IN MATHEMATICS"
300  VTAB 11: PRINT  TAB( 8)"VOLCANIC ERUPTION FALLOUT": PRINT : PRINT :
      PRINT  TAB( 18)"* * *": PRINT : PRINT : PRINT  TAB( 13)"FALLOUT TA
     BLES"
310  VTAB 20: HTAB 7: INPUT "PRESS <RETURN> TO CONTINUE ";A$
320  HOME : VTAB 2: PRINT "THIS PROGRAM CALCULATES THE PATTERN OF"
330  VTAB 4: HTAB 2: PRINT "FALLOUT, REYNOLDS NUMBERS AND DRAG"
340  VTAB 6: HTAB 6: PRINT "COEFFICIENTS FOR A VOLCANO"
350  FOR I = 1 TO 9
360  READ RHO(I): NEXT I
370  FOR I = 1 TO 9
380  READ ETA(I): NEXT I
390  FOR I = 1 TO 9
400  READ WS(I): NEXT I
410  GOTO 430
420  HOME
430  VTAB 10: PRINT "ENTER THE DIAMETER OF AN"
440  VTAB 12: INPUT "AVERAGE PARTICLE IN CENTIMETERS: ";D
445  IF D <  = 0 THEN  PRINT : PRINT "DIAMETER MUST BE POSITIVE.": GOTO
     430
450  VTAB 16: PRINT "ENTER THE DENSITY OF AN AVERAGE"
460  VTAB 18: INPUT "PARTICLE IN GRAMS PER CUBIC CM: ";SIGMA
465  IF SIGMA <  = 0 THEN  PRINT : PRINT "DENSITY MUST BE POSITIVE.": GOTO
     450
470  FOR I = 1 TO 9
480 TM(I) = (D ^ 2 * SIGMA * G) / (18 * ETA(I))
490 T(I) =  INT (100 * TM(I) + .5) / 100
500  NEXT I
510  REM
520  REM   CHOOSE OPTIONS
530  REM
540  HOME : VTAB 6: HTAB 3
550  PRINT "DO YOU WANT TO:"
560  VTAB 8: HTAB 6: PRINT "A) PRINT THE FALLOUT TABLES"
570  VTAB 10: HTAB 6: PRINT "B) PRINT THE TABLE OF REYNOLDS"
580  HTAB 9: PRINT "NUMBERS AND DRAG COEFFICIENTS"
590  VTAB 13: HTAB 6: PRINT "C) ENTER NEW DATA"
600  VTAB 15: HTAB 6: PRINT "D) QUIT"
```

```
610  VTAB 19: HTAB 3: PRINT "ENTER THE LETTER OF YOUR"
620  HTAB 6: INPUT "CHOICE AND PRESS RETURN:  ";A$
630  IF A$ = "A" GOTO 680
640  IF A$ = "B" GOTO 1130
650  IF A$ = "C" GOTO 420
660  IF A$ = "D" GOTO 1340
670  GOTO 520
680  REM
690  REM   FALLOUT TABLES
700  REM
710  HOME : HTAB 11: VTAB 3
720  PRINT "PATTERN OF FALLOUT"
730  HTAB 11: PRINT "THROUGH EACH LAYER": PRINT
740  PRINT "ALT." TAB( 6)"TERMINAL" TAB( 15)"TIME OF" TAB( 24)"WIND" TAB(
     30)"HORIZONTAL"
750  PRINT  TAB( 2)"KM" TAB( 6)"VELOCITY" TAB( 17)"FALL" TAB( 24)"SPEED"
      TAB( 31)"DISTANCE"
760  PRINT  TAB( 8)"CM/S" TAB( 17)"HRS" TAB( 24)"KM/HR" TAB( 34)"KM": PRINT

770  FOR K = 1 TO 9:I = 10 - K
780 H = 100000 / (TM(I) * 3600)
790 HR =  INT (H * 100 + .5) / 100
800 W = 1.852 * WS(I)
810  IF W > 99.99 THEN X = 24
820  IF W < 99.99 THEN X = 25
830 HD =  INT (10 * H * W + .5) / 10
840 W2 =  INT (100 * W + .5) / 100
850  PRINT I - 1"-"I TAB( 8)T(I) TAB( 17)HR TAB( X)W2 TAB( 33)HD
860  NEXT K
870  VTAB 22: INPUT "PRESS RETURN TO CONTINUE ";A$
880  HOME : HTAB 11: VTAB 3
890  PRINT "PATTERN OF FALLOUT"
900  HTAB 4: PRINT "FROM EACH ALTITUDE TO SEA LEVEL": PRINT
910  PRINT "ALTITUDE" TAB( 13)"TIME OF FALL" TAB( 28)"HORIZONTAL"
920  PRINT  TAB( 4)"KM" TAB( 17)"HRS" TAB( 29)"DISTANCE"
930  PRINT  TAB( 32)"KM": PRINT
940 TH(1) = 0:TD(1) = 0
950  FOR I = 1 TO 9
960 H = 100000 / (TM(I) * 3600)
970 TH(I) = TH(I - 1) + H
980 HR =  INT (H * 100 + .5) / 100
990 W = 1.852 * WS(I)
1000 HD =  INT (10 * H * W + .5) / 10
1010 TD(I) = TD(I - 1) + HD
1020  NEXT I
1030  FOR K = 1 TO 9:I = 10 - K
1040 TF =  INT (100 * TH(I) + .5) / 100
1050  IF TF > 9.9 THEN X = 16
1060  IF TF < 9.9 THEN X = 17
1070  IF TD(I) > 999.9 THEN Y = 30
1080  IF TD(I) < 999.9 THEN Y = 31
1090  PRINT  TAB( 3)I - 1"-"I TAB( X)TF TAB( Y)TD(I)
1100  NEXT K
1110  VTAB 23: INPUT "PRESS RETURN TO CONTINUE ";A$
1120  GOTO 520
1130  REM
1140  REM  TABLE OF REYNOLDS NOS.
1150  REM  AND DRAG COEFFICIENTS
1160  REM
1170  HOME : VTAB 4
1180  PRINT  TAB( 3)"ALTITUDE" TAB( 15)"REYNOLDS" TAB( 29)"DRAG"
```

```
1190  PRINT  TAB( 6)"KM" TAB( 16)"NUMBER" TAB( 26)"COEFFICIENT": PRINT
1200  FOR K = 1 TO 9:I = 10 - K
1210 R = (D * TM(I) * RHO(I)) / ETA(I)
1220 RE =  INT (100000 * R + .5) / 100000
1230 DC =  INT (10 * 24 / R + .5) / 10
1240  PRINT  TAB( 4)I - 1" - "I TAB( 15)RE TAB( 28)DC
1250  NEXT K
1260  VTAB 21: INPUT "PRESS RETURN TO CONTINUE ";A$
1270  GOTO 520
1280  REM    DENSITY DATA
1290  DATA  .000123, .000109, .000097, .000087, .000080, .000069, .00006
     1, .000052, .000046
1300  REM    VISCOSITY DATA
1310  DATA   .000166, .000163, .000161, .000158, .000155, .000153, .0001
     50, .000149, .000148
1320  REM    WIND SPEED DATA
1330  DATA   11,17.5,28,36,51,54.5,58.5, 58, 50.5
1340  END
```