A Backwater Curve for the Windsor Locks Canal

Teacher Resource Book

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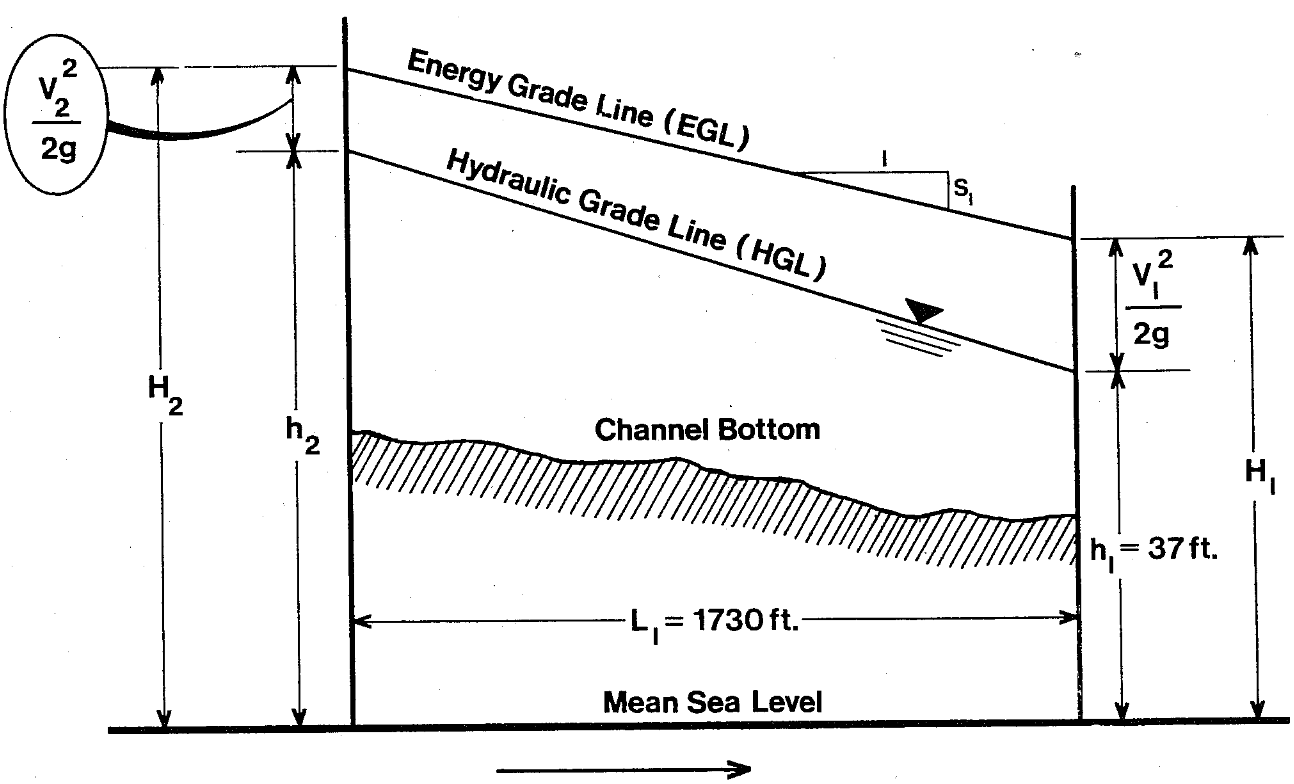

Title: A Backwater Curve for the Windsor Locks Canal

Source: R. Stevens Kleinschmidt, President, Kleinschmidt Associates, Pittsfield, Maine

Prerequisites: High School, Algebra I

Skills Needed: Algebraic operations; geometry (area and perimeter of a trapezoid, slope of a line, similar triangles); hand calculator; computer is optional

Summary: Kleinschmidt Associates are undertaking an analysis of the Windsor Locks Canal to determine whether it can be used in the production of electricity. The question to be answered is "What upstream water surface elevation would be necessary to deliver 1,500 cu ft of water per second to the downstream end of the canal?"

Comments: Necessary formulas are provided. Detailed background information is not required. The interest of the problem lies in the use of simple mathematical skills to solve the problem by the method of successive approximation.

Suggested Classroom Uses.

A unit in a mathematics course

Independent Study

A project for the Math Club

Enrichment in Geometry (Student Resource Book, Section V, the Problem, Part A; Teacher Resource Book, Section VIII, Additional Questions, parts A and C)

Enrichment in Trigonometry (Teacher Resource Book, Section VIII, Additional Questions, part B)

Enrichment in Calculus (Teacher Resource Book, Section VIII, Additional Questions, part B)

Experience in Computer Simulation (Teacher Resource Book, Section VIII, Additional Questions, Part D)

A Backwater Curve for the Windsor Locks Canal

Teacher Resource Book

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Table of Contents

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I. Introduction

This book is the Teacher Resource Book for the industry-related problem, "A Backwater Curve for the Windsor Locks Canal." It is one part of a coordinated package of materials called an AIM Learning Module. The total AIM module consists of: Video I, "The Problem"; Video II, "Problem Preparation"; the Student Resource Book; the Teacher Resource Book; two microcomputer BASIC programs; and Video III, "A Solution."

All three Videos are on-site conversations with the industrial representative who has actually addressed this problem at one time in his professional life. In Video I, the discussion centers on the problem and how it arose. In Video II the industrial representative provides hints and direction to assist the student in making progress toward a solution. The discussion in this video also gives the student information about the career of civil engineering and in particular hydraulic engineering. In Video III the industrial representative provides the solution of the problem as he solved it while on the job.

This problem is intended for use in any mathematics course above Algebra I. The mathematics required is: simple algebraic operations, slope of a line, area of a trapezoid, the Pythagorean Theorem. The interest of the problem lies in the way these skills are put together to solve a real-world problem. A hand calculator is essential. Access to a computer is desirable.

An important aspect of each AIM Learning Module is its flexibility as a teaching tool completely under the control of the teacher. Section II, Teaching Strategies, lists some of the many ways in which the teacher can make use of this material.

The problem is set out in ten parts in the Student Resource Book, Section V, in order to make it easier to use with students of different ability levels. A detailed solution of each part of the problem is presented in this book in Section V, A Solution in Detail. At the end of each of the ten parts, comments are included to help answer questions which might arise.

Section III, Where to Begin, makes some suggestions about initial activities which might help the students get started. For students who have little experience with anything but short-answer work, getting started is the first hurdle.

Section IV, Developmental Approach, sets forth the thought processes through which a student might proceed in reaching a solution to the problem. One way of formalizing your own process of thinking is to ask yourself a series of questions. With this in mind, the Developmental Approach is written completely in qestion format. The questions are divided into 12 sections. Each section begins with a lead question identifying a general area of concern. This leading question is followed by several more detailed questions which guide the student in small steps to an appropriate answer for the leading question.

The Developmental Approach can serve two primary purposes. It can be used by the teacher as a source of leading questions to guide student discussion of the problem in a class. Or it can be given to the students to help them in their discussions. If the problem is to be used as independent study this section will be especially useful to help spur the student's thinking. The questions are written in the first person to help the individual student identify them as part of his/her own creativity. It is probable that for a student's first experience with an AIM module the Developmental Approach will be important to the student. After the student has worked through one or two of the AIM modules, the student will have acquired enough problem-solving technique to make the Developmental Approach unnecessary.

Section VI discusses how to use computer programs available on the AIM diskette. One is interactive and the other proceeds directly to the solution. In Section VII a brief description of a student report is given and a suggested format is included in the appendix.

Section VIII consists of a number of additional questions and projects which are related to this AIM module and the techniques it requires. Some of these are "What if" questions which give the student a chance to explore by computer what would be the result of changing some of the constants given in the problem.

II. Teaching Strategies

There are many exciting ways in which the AIM materials can be used in the high school curriculum. You are, of course, free to use your creativity to modify these and to devise others that fit your individual situation.

Whatever method you choose, the goal is twofold: to have the students experience mathematics in an industrial setting; to raise the student's awareness of careers in mathematics.

A. A Unit in a Mathematics Course

The objective is to involve the student in a discovery-learning approach geared to developing and sharpening the following skills:

1) reasoning and model building

2) real-world problem solving

3) communicating verbally about mathematics

4) writing technical material concisely and accurately

5) making use of resource materials

6) using the computer in problem solving

Step 1. Preparation. Give each student a copy of the Student Resource Book. (Permission is granted to copy these for classroom use.) The student reads it and becomes acquainted with the problem and its setting.

Step 2. Video I. The students view Video I. Through this on-site video they meet the industrial representative who discusses the problem and some related insights.

Step 3. Getting Started. The students study the problem, become familiar with the terms and consult reference books as needed. They work the preliminary activities, Section A of the problem, and begin to develop a mathematical model that provides the context in which the problem can be solved. They talk about the problem, its setting, the technical terms, the assumptions made, and ways in which they might attack the problem. When they feel the need for a little extra direction, they view Video II. At this time the hints from the industrial representative will be meaningful for them.

Step 4. Creating a Solution. The students work as a class or as individuals and discuss their work. At this stage you, as teacher, skillfully nudge the students toward a solution. To help you accomplish this, Section IV of this book, entitled Developmental Approach, provides a succession of questions some of which you might use to stimulate discussion. A complete solution of the problem is provided in Section V. However, the teacher's role is not to provide a solution but rather to encourage and tease the students to find their own solution. Students interact with their peers and their instructor and also use the resources of their campus (library, computer center, faculty, etc.). Students come to class ready to report on their mathematical progress. When solutions are obtained, they present their solutions and field questions on their work.

Step 5. The Computation. Two solution programs are provided on a Diskette for use with Apple II. One is an interactive program that allows the student to follow the solution in a step-by-step way, making appropriate decisions when necessary. Each student should use this program at least once. The second program is also user-friendly but goes directly to the solution of the problem once the initial data is entered. This program allows the student to answer "What if" questions which are likely to arise. Some such questions are suggested in Section VIII. Students will think of many on their own. Some may wish to write their own solution program.

Step 6. The Report. Each student then writes a technical report on the problem and its solution. The report is discussed in Section VII and a suggested format is given in Section X.

Step 7. The students view Video III, "A solution." At this time they can compare their solution with that provided on the Video.

B. A Unit for Independent Study

The method described in A is an "ideal" way to use the AIM materials in a class-oriented problem-solving situation. The same general method is equally effective when used as individual instruction or independent study for one or more students. In such a case class discussion gives way to periodic teacher conferences. If the student is short of time or unable to proceed, the teacher can provide the list of questions given in the Developmental Approach (Section IV) to lead the thought processes of the student.

C. Enrichment in a Variety of Courses

Use AIM videos I, II and III as a lecture presentation to a class when you wish to stimulate interest in mathematics by demonstrating an application of the material they are studying.

Assign parts of the problem when the class work deals with some skill used in the problem solution. In this case you might show AIM Video I to acquaint the class with the problem setting and then give a brief discussion of the method of solution.

Students can experience computer simulation by using the computer solution with different data. The "What if" problems suggested in Section VIII illustrate the use of the computer to answer questions about a physical situation.

D. Project for a Math Club

The aim materials make an exciting series of programs for a math club. The technique described in Section A can easily be adapted to this setting. The fact that math clubs include students at various levels adds interest in the sharing of skills.

E. Developing Career Awareness

All three videos give a first hand picture of a civil engineer at work. Implicitly all three raise the student's consciousness of civil engineering as a career and of the importance of mathematics in his/her future career choices. Much of the discussion in The Problem Preparation cassette is devoted to hydraulic engineering in particular.

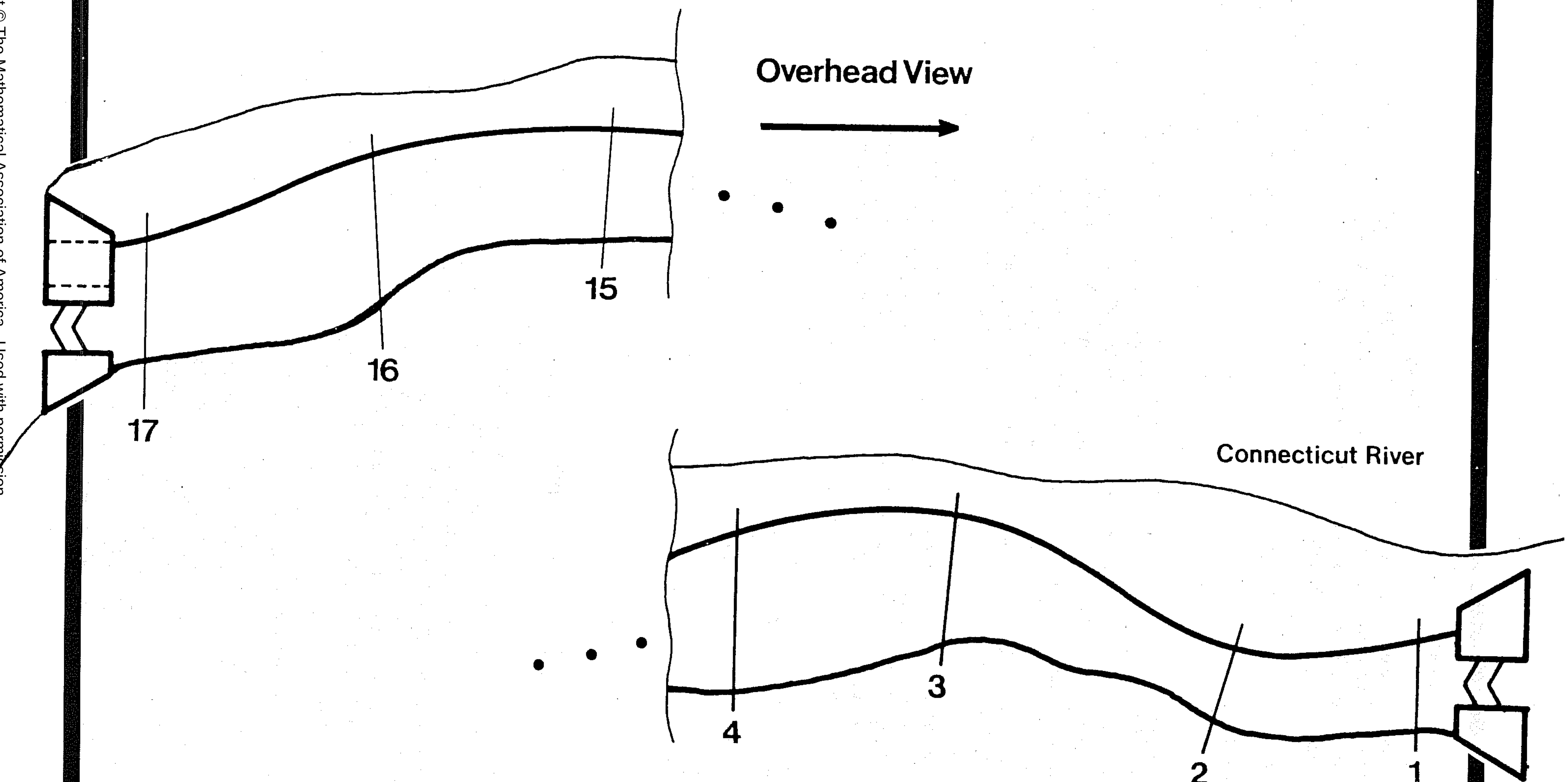
F. Group Presentations

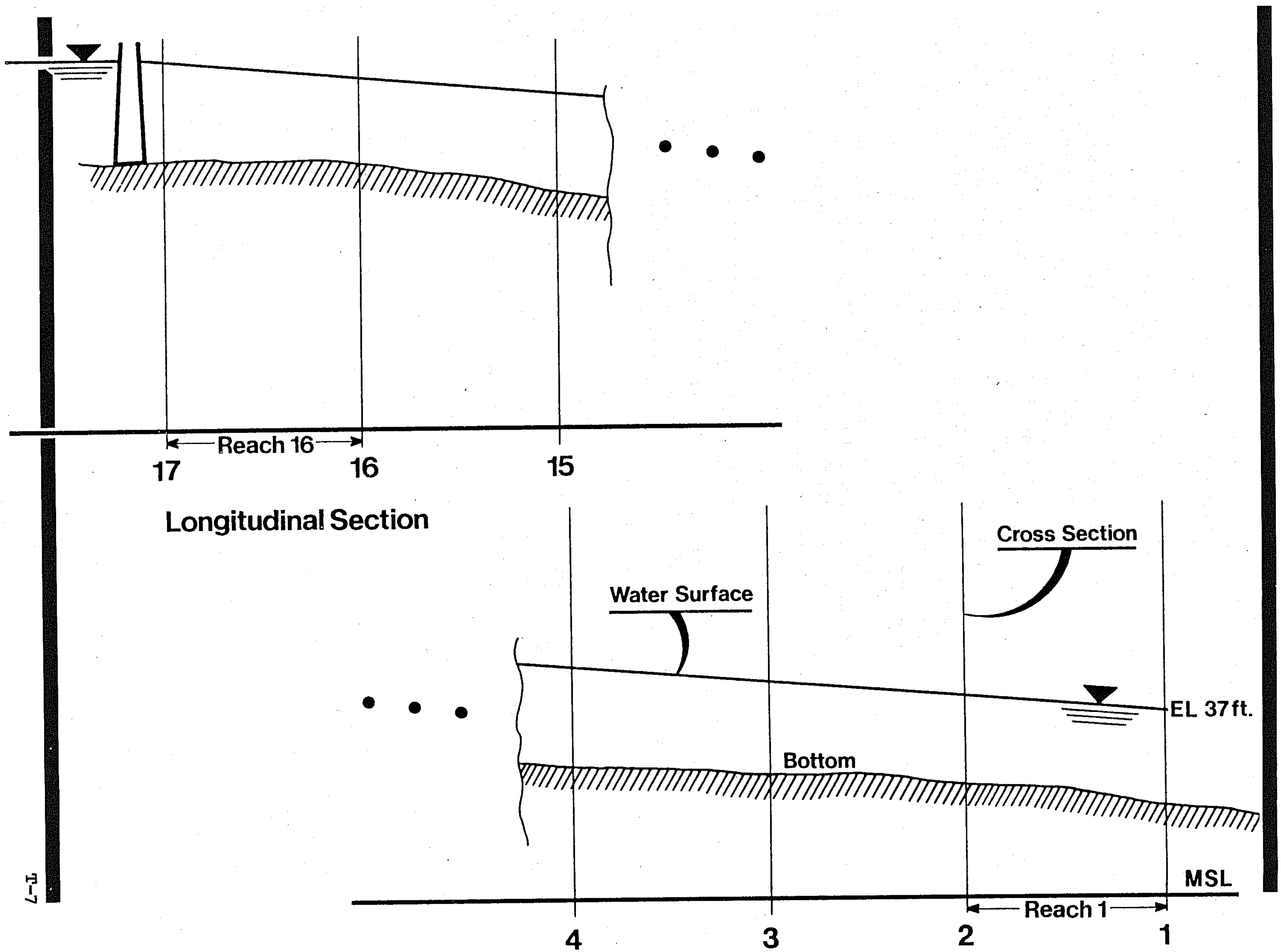
Use the AIM Video as the basis of a presentation at a regional Meeting of NCTM.

Use AIM Video and written materials as a resource for a workshop for your area group of mathematics teachers.

FIGURE 1

Plan of Windsor Locks Canal




Reach 16
17
16
15
Longitudinal Section
Cross Section
Water Surface
EL 37 ft.
Bottom
MSL
Reach 1
4
3
2
1

III. Where to Begin

A major hurdle in problem solving is deciding what to do first. Students in particular are accustomed to seeing the answer at the first step, and are therefore reluctant to do anything till they are sure it will lead to the answer. In this problem the first requirement is to understand the setting and the notation. There are a few obvious things to do immediately, for example, reading and rereading the Student Resource Book and watching Video I. It may increase their confidence if they read about water power in an encyclopedia. Have them locate the Connecticut River and Windsor Locks on a map.

Have the students make a list of important words, the symbols that are introduced, and formulas as they occur. For your guidance, Section X, Appendix, has a sample glossary and list of notation and formulas. Use this to suggest words to be sure the student list is complete. However, do not give them the formulas. Only the first four are given to them in sections II and III of the Student Resource Book. The students add others to these as they develop them in the course of working the problem. Part of the experience is finding the formulas in the reading and developing others as they are required.

Encourage the students to do the Preliminary Activities, Part A of the Problem in Detail, along with the reading. These are designed to help them understand the description of the canal and to familiarize them with the basic calculations. Part B should also be discussed and compared with Figure 1.

Students can do the calculations in D immediately, since these come directly from the formulas. Depending on the type of hand calculator they use, they may need help in calculating the cube root of R^4. Part C asks the student to generalize the numerical work in A(4) to produce two formulas which will be useful in the later calculations. Generalizing a numerical calculation to a formula should be a good experience for them. Encourage them to use their work in A(4) which should be equivalent to Figure 15 of the Teachers Resource Book.

When the problem has been carried to the end of Part D, the "guessing game" begins. This is a new problem-solving experience, and FUN.

IV. Developmental Approach

1. Am I familiar with the terms?

   Have I read the background several times?
   Have I made a list of new words?
   Can I identify reach, section, HGL in Fig. 1?
   Do I understand the difference between EGL and HGL in Figure 5?
   Have I made a list of notation and formulas?
   Have I plotted the canal bed in Part A?
   Can I divide the section area into simpler shapes for calculating area?
   What is the area of a trapezoid?
   What is the area of a triangle?

2. Do I understand the table of data?

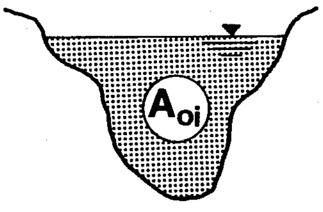
   What quantities are given?
   What are A_{02}, P_{02}, and W_2?
   Are my answers in Problem A close to these?

3. Do I understand the calculations?

   What quantities need to be calculated?
   What are the formulas given?
   How do I calculate S from the formula given?
   Will it be easier if I solve for S?
   In what order should I do the calculations?
   Does it matter?

4. What changes when the water surface rises?

   What is h at Section 1?
   Does h change at this section?
   What value is used for h in the table?
   If h at section 2 is 39, what is Δh_2?
   If h changes by Δh what is the new h?
   Does changing h change A and P?
   What assumption do I make about W?
   When I draw on the sketch a rectangle of water to represent this change what are the dimensions of the rectangle?
   What is the area of the rectangle?
   At Section i what do I add to A_{0i} to get A_i?
   How much does the wetted perimeter change?
   What do I add to P_{0i} to get P_i?



5. Can I do all the calculations at Section 1?

   What does the table give for A_1, P_1, W_1?

   What do the formulas give for V_1, R_1, A_1, H_1?

6. How much does h change at section 2?

Am I told how much?
Have I a formula to calculate how much?
Is the HGL a straight line?
If it were a straight line what would I need to know to calculate h_2?
What is the distance between Sections 1 and 2?
Do I know the slope of the HGL at Section 1?
What line do I know the slope of?
Are the HGL and the EGL parallel?
Are they near enough so I could use S_1 as the slope of the HGL is making a first guess?
Slope = rise/run. What is the run in this case?
What is the rise?
What is a good guess for h_2?
Can I write this as a formula in terms of S_1, L_1, and h_1?

7. Can I do the calculations at Section 2?

What do I find from the table?
How do I find A_2 and P_2?

Can I calculate V_2, R_2, S_2, and H_2?
Are these values exactly correct?
Is there another method to calculate H_2?
What is the average slope of the EGL in Reach 1?
How can I calculate H_2 from H_1?
What is H_2'?

8. How good was my guess?

What two values of H_2 did I find?
What is $|H_2 - H_2'|$?
Is this small enough? If so I can go to Section 3.
If not, what is a better guess?

9. How can I make a better guess?

Which is bigger, H_2 or H_2'?

Since $H_2 = h_2 + V^2/2g$ what should I do to h_2 to make H_2 smaller?
How much smaller should h_2 be?
What number might be a good guess?
Can I write a formula for my choice in case I need to use it again?

10. What are the calculations with the new h_2?

How do I find A_2, P_2?

Now what are V_2, R_2, S_2, and H_2?

What is the new average slope for the EGL in Reach 1?
What is the new H_2'?

11. How good is my second guess for h_2?

Can I answer this like I did in 8?
If h_2 is good enough, what values should I assign to h_2, H_2, and S_2?

12. What about the rest of the Problem?

Where should I start for Section 3?
What Reach am I talking about?
In step 6 I wrote h_2 in terms of h_1, L_1, and S_1. What should I write to go from Section 2 to Section 3?
What calculations do I make when I repeat Step 7?
What h will I be fixing at Step 11?
What section do I do next?
What is the last h_i I need to find?

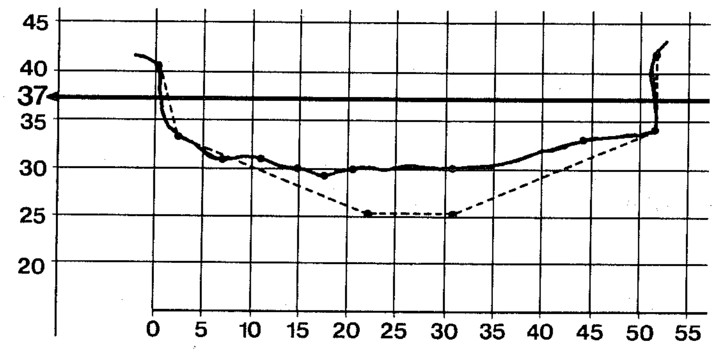
Now that I have devised a method I can go to the interactive computer program to carry it out. (Or I could write my own program.)

V. A Solution in Detail

In this section a detailed solution is presented. It follows the thought processes outlined in Section IV, Developmental Approach. Each step in the calculation is followed by comments which reflect some of the questions that might arise.

A. Preliminary Activities

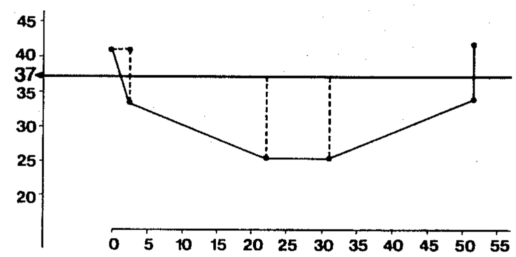
FIGURE 2



Plot of Canal Bed

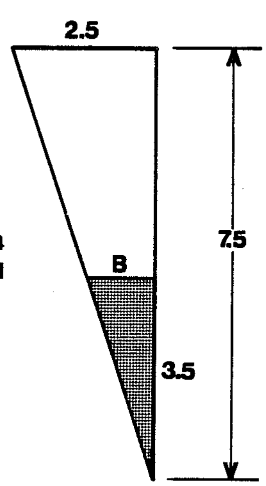
1. and 2. The results of the plotting in 1 and 2 are shown in Figure 2. Figure 3 shows the additional lines that divide the area into trapezoids and triangles. Figure 4 is an enlargement of the triangular section and the larger triangle similar to it.

FIGURE 3



Calculation of Section Area

FIGURE 4



The Section Area

The triangle:

Height of triangle = 37 - 33.5
= 3.5 ft.

Base of triangle is found from similar triangles:

$$\frac{\text{Base}}{3.5} = \frac{2.5}{7.5}$$

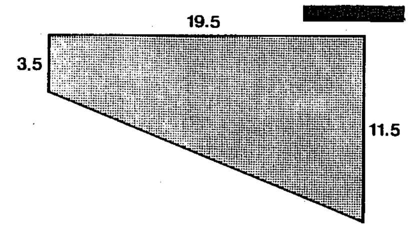
Base = 1.2.

Area of triangle
= (1/2)(base)(height)
= (1/2)(1.2)(3.5)
= 2.1 sq ft.

The Left-hand Trapezoid:

Length of left base = 3.5 ft.
Length of right base = 37 - 25.5 = 11.5 ft.
Height = 22 - 2.5 = 19.5 ft.
Area = 1/2(sum of bases)(height) = 1/2(3.5 + 11.5)(19.5) = 146.3 sq ft.

FIGURE 5



The Rectangle:

Length = 31 - 22 = 9 ft.
Width = 11.5 ft.
Area = (length)(width) = (9)(11.5) = 103.5 sq ft.

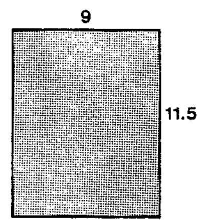
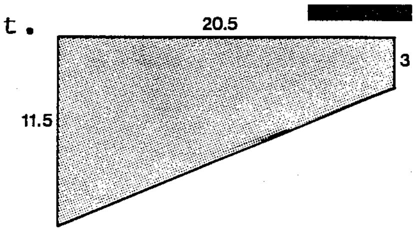


FIGURE 6

The Right-hand Trapezoid:

Length of left base 11.5 ft.
Length of right base 37 - 34 = 3 ft.
Height = 51.5 - 31 = 20.5 ft.
Area = 1/2(11.5 + 3)(20.5) = 148.6 sq ft.

FIGURE 7



Total area **A** = 2.1 + 146.3 + 103.5 + 148.6
A = 400.5 sq ft.

<u>The Wetted Perimeter</u>

The Triangle:

The hypotenuse, c, of a right triangle with sides a, b, c is $c = \sqrt{a^2 + b^2}$.
In this case a = 1.2, b = 3.5
Hypotenuse $= \sqrt{(1.2)^2 + (3.5)^2}$ = 3.7 ft.

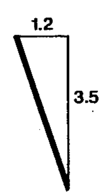
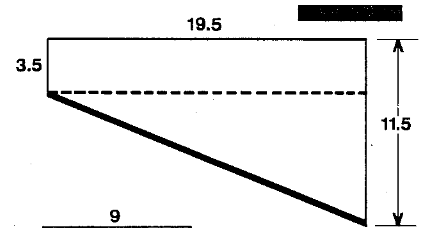


FIGURE 8

The Left Trapezoid:

Side a = 19.5 ft.
Side b = 11.5 - 3.5 = 8 ft.
Hypotenuse $= \sqrt{(19.5)^2 + (8)^2}$ = 21.1 ft.

FIGURE 9



The Rectangle:

Width = 9 ft.

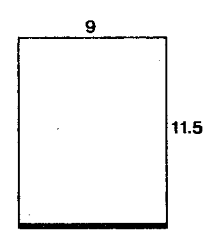


FIGURE 10

The Right Trapezoid:

Side a = 20.5 ft.
Side b = 11.5 - 3 = 8.5 ft.
Hypotenuse $= \sqrt{(20.5)^2 + (8.5)^2}$ = 22.2 ft.
The Right base = 3 ft.

Wetted Perimeter = 3.7 + 21.1 + 9 + 22.2 + 3
P = 59.0 ft.

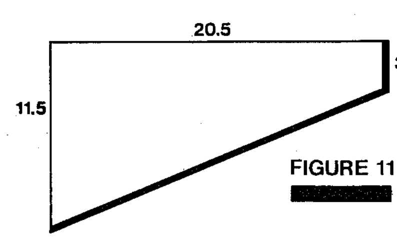


FIGURE 11

Water Surface Width

The left hand edge of the water surface is 2.5 - 1.2 = 1.3 on the horizontal scale and the right edge is 51.5.

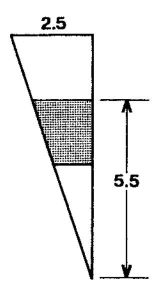
W = 51.5 - 1.3
= 50.2 ft.

Hydraulic Radius

R = A/P
= 400.5/59
= 6.8 ft.

3. If the water level rises to 39 ft, the height of the triangular section changes and the base also changes.

FIGURE 12



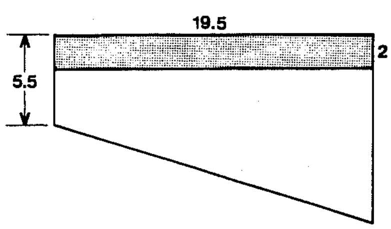
The Triangle:

New height is 39 - 33.5 = 5.5 ft.

$$\frac{\text{New Base}}{5.5} = \frac{2.5}{7.5}$$

Base = 1.8 ft.
Area of triangle = 1/2(1.8)(5.5)
= 5.0 sq ft.
Increase in area = 5.0 - 2.1
= 2.9 sq ft.

FIGURE 13



Left Trapezoid:

Increase in area is a rectangle of width 2 ft and length 19.5 ft.
Increase in area = 2 x 19.5
= 39 sq ft.

Rectangle:

Increase in area is a rectangle of width 2 ft and length 9 ft.
Increase in area = 2 x 9 = 18 sq ft.

Right Trapezoid:

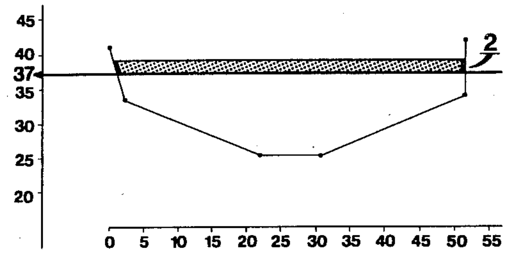
Increase in area = 2 x 20.5
= 41 sq ft.

Total section area for 39 ft water surface elevation
= Section area for 37 ft + increase in area
= 400.5 + (2.9 + 39 + 18 + 41)
A = 501.4 sq ft.

The Wetted Perimeter

The wetted perimeter is changed only by the increased length of the left side and the right side. The sides of the triangle are now 1.8 ft and 5.5 ft so the hypotenuse = 5.8 ft. The increase in length is 5.8 - 3.7 = 2.1 ft.

The increase in length at the right hand side is 2 ft. (See Figure 14.)



Area and Perimeter Changes (Actual)

FIGURE 14

Wetted Perimeter for 39 ft water surface elevation is wetted perimeter for 37 ft plus the increase.

P = 59.0 + (2.1 + 2)
  = 63.1 ft.

The Water Surface Width:

The lefthand edge is now at 2.5 - 1.8 = 0.7 on the horizontal scale. The right edge is not changed.

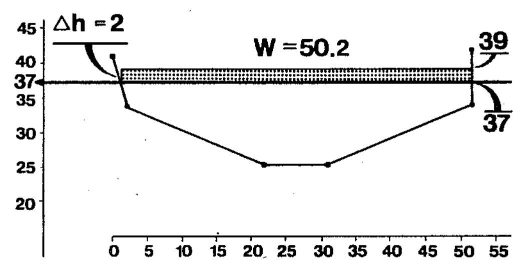
Water Surface width for 39 ft water surface elevation is

W = 51.5 - 0.7
  = 50.8 ft.

Hydraulic Radius

R = A/P
  = 501.4/63.1
  = 7.9 ft.

4. In this case W does not change, W = 50.2 ft. (See Figure 15.)



Area and Perimeter Changes Rectangular Approximations

FIGURE 15

Increase in section area is area of a rectangle of length 50.2 and width 2.

Increase in section area = 2(50.2)
  = 100.4 sq ft.

Section Area at 39 ft ≈ 400.5 + 100.4 sq ft
  = 500.9 sq ft.

Wetted perimeter increases by 2 ft on each side.

Wetted perimeter ≈ 59.0 + 4
  = 63.0 ft.

R = 500.9/63.0
  = 8.0 ft.

The accuracy of the approximation is best demonstrated as a percent of the figure being calculated:

$$\text{Percentage error} = \frac{\text{Error}}{\text{Quantity}} \times 100 .$$

The following table answers the last part of A(4).

Quantity	Exact Value	Approximate Value	Error	Percentage Error
A	501.4	500.9	0.5	0.1 %
P	63.1	63.0	0.1	0.2 %
W	50.8	50.2	0.6	1.1 %
R	7.9	8.0	0.1	1.3 %

Comment:

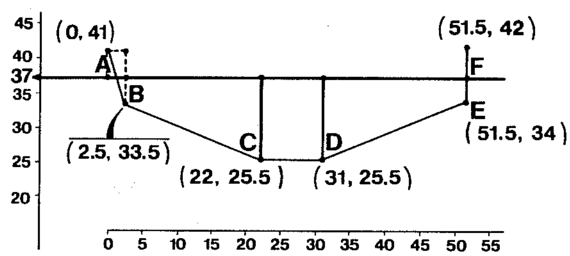
The method of calculation described for problem A makes no use of a coordinate system. For a student in a geometry class this method is closely related to their current work and reinforces geometric ideas.

Students currently in Algebra could easily use coordinates if desired. This approach is outlined here.

Coordinate Approach:

The x axis is mean sea level.
The y axis is at the left edge of the canal. (See Figure 16.)
The coordinates of A must be calculated.

FIGURE 16



Coordinate Representation of Cross Section

Formulas Needed:

Slope of a line joining (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of a line with slope m and y-intercept b is $y = mx + b$.

Distance between the points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Calculations:

Slope of $\overleftrightarrow{AB}$ is $\frac{33.5 - 41}{2.5 - 0} = -3$.

Equation of $\overleftrightarrow{AB}$ is $y = -3x + 41$.
At A, $y = 37$, so $37 = -3x + 41$
$x = 1.3$.

Area is in four parts

$A_1 = (1/2)(2.5 - 1.3)(37 - 33.5) = 2.1.$
$A_2 = (1/2)(37 - 33.5 + 37 - 25.5)(22.0 - 2.5)$
$= 146.3.$
$A_3 = (31 - 22)(37 - 25.5) = 103.5.$
$A_4 = (1/2)(37 - 25.5 + 37 - 34)(51.5 - 31.0)$
$= 148.6.$

Section Area = 400.5 sq ft.

To find the wetted perimeter, use the distance formula to get the length of each segment.

Length $\overline{AB}$ =
$\sqrt{(2.5 - 1.3)^2 + (33.5 - 37)^2} = 3.7$ ft.
Length $\overline{BC}$ =
$\sqrt{(22.0 - 2.5)^2 + (25.5 - 33.5)^2} = 21.1$ ft.
Length $\overline{CD} = 31.0 - 22.0 = 9$ ft.

Length $\overline{DE}$ =
$\sqrt{(51.5 - 31.0)^2 + (34.0 - 25.5)^2} = 22.2$ ft.
Length $\overline{FE} = 37 - 34 = 3.$

Total Wetted Perimeter P = 59 ft.

B. The Given Data

The student is required only to study the table. It might be well to have students check this by asking each other questions like "what is the value of A_{05}?"

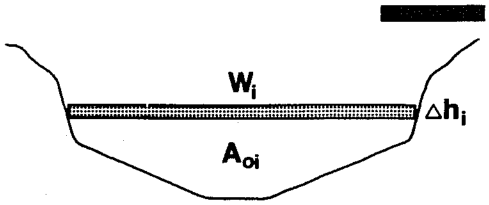
The values of A_{02}, P_{02}, W_2 calculated in part A do not quite agree with the values given us in the table. No doubt Dr. Kleinschmidt chose to use integral values when the set up the table. Students might discuss how accurate it is reasonable to expect the workers to be in making improvements on a canal bed.

C. Adjust the Data

1. First draw a cross section of the canal. (See Figure 17.)

   Label the area A_{0i}.
   Shade in the rectangle that represents increased area.
   Mark the dimensions on the rectangle.
   Area of the rectangle is $W_i \; \Delta h_i$.

FIGURE 17

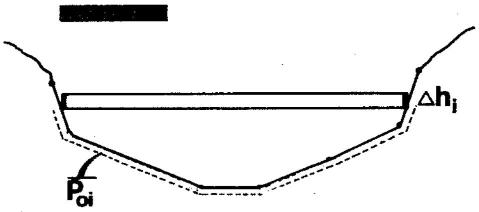


Change in A

Adjusted area of Section i is

$$A_i = A_{0i} + W_i \Delta h_i.$$

FIGURE 18



Change in P

Draw a similar figure, mark P_{0i} and darken the approximate increase in wetted perimeter. (See Figure 18.) Increase is $2\Delta h_i$.

$$P_i = P_{0i} + 2\Delta h_i.$$

2. In A(4), $\Delta h_2 = 39 - 37 = 2$.

The calculation in A(2) is $A_{02} = 400.5$ sq ft.
$W_2 = 50.2$ ft.

According to the formula in C(1)
$A_2 = 400.5 + (50.2)(2) = 500.9$ sq ft.

Answer in A(4) is $A_2 = 500.9$ sq ft.

According to the formula
$P_2 = 59 + 2(2) = 63$ ft.

Answer in A(4) is 63 ft.

Comment:

The formulas developed here will be used again and again. Have the students add them to their formula list.

D. Calculate the Data for Section 1

$A_1 = A_{01} = 433$ sq ft
$P_1 = P_{01} = 64$ ft
$W_1 = 60$ ft

given data

$V_1 = Q/A_1 = 1500/433 = 3.4642$ ft/sec

$R_1 = A_1/P_1 = 433/64 = 6.7656$ ft

The formula $V = \frac{1.49}{n} R^{2/3} S^{1/2}$ is not in a convenient form.

Solve for S:

$$n V = 1.49 R^{2/3} S^{1/2}$$

$$\frac{n V}{1.49 R^{2/3}} = S^{1/2}$$

$$S = \frac{n^2 V^2}{(1.49)^2 R^{4/3}}.$$

Since n = 0.022 throughout,

$$\frac{n^2}{(1.49)^2} = 0.0002180$$

$$\mathbf{S = \frac{0.0002180\ V^2}{R^{4/3}}}$$

$$S_1 = \frac{(0.000218)(3.4642)^2}{(6.7656)^{4/3}}$$

$$= 0.0002045 \text{ ft/ft.}$$

$$H_1 = h_1 + V_1{}^2/2g$$

$$= 37 + \frac{(3.4642)^2}{2(32.2)}$$

$$= 37 + 0.1863$$

$$= 37.1863 \text{ ft.}$$

Comment:

The order of calculation makes little difference as long as V is calculated before S or H.

Students may wish to calculate S without first solving for S. This can certainly be done:

$$3.4642 = \frac{1.49}{0.022}(6.7656)^{2/3} S^{1/2}$$

$$0.014299 = S^{1/2}$$

$$S = 0.0002045.$$

It is much better to solve algebraically for S. They should do this and add it to their list of formulas.

The better students may well question the accuracy of these and following calculations. Because the slopes involved are quite small and the changes in slope are slight, it is necessary to assume that there are at least four significant digits beyond the decimal point in S. The values of h and H must be calculated to at least four decimal places. The tolerance of 0.005 assigned requires this.

E. The Data for Section 2

1. Since HGL is approximately parallel to the EGL and the EGL has slope S_1 we can use S_1 as a first guess for the slope of the HGL,

$$\frac{\text{Rise}}{\text{run}} = S_1 \text{ or } \frac{h_2 - h_1}{L_1} = S_1$$

$$\text{Rise} = (\text{run})(S_1) \text{ or}$$

$$\mathbf{h_2 = h_1 + S_1 L_1}$$

In Section 1, the run = 1730 ft,

$S_1 = 0.0002045$ ft/ft,

rise = (0.0002045)(1730)
= 0.3538 ft.

Guess for h_2 = 37.3538 ft.

2. Calculation of A_2 and P_2:

$A_{02} = 400$, $W_2 = 50$, $\Delta h_2 = 0.3538$

$A_2 = A_{02} + W_2 \Delta h_2 = 400 + 17.6900$
$= 417.6900$ sq ft.

$P_{02} = 60$

$P_2 = P_{02} + 2\Delta h_2 = 60 + 0.7076$

$= 60.7076$ ft.

Comment:

The students may need some encouragement in making this guess. The questions in the developmental approach may help here. The first thing they need to realize is that the HGL is not a straight line but it is "almost" one. Second if you know the slope and one point on a line you can calculate any point on the line given the horizontal distance. Third they need to guess that the slope of the EGL could be used for the HGL and would give a good guess for h_2. Mathematics students are not accustomed to guessing especially in getting a measurement. This will be good experience for them.

F. Calculate the Data for Section 2

1. We now have some straightforward calculations:

$h_2 = 37.3538$
$A_2 = 417.6900$
$P_2 = 60.7076$
$V_2 = 3.5912$
$R_2 = 6.8804$

$$S_2 = \frac{(0.0002180)(3.5912)^2}{(6.8804)^{4/3}}$$

$S_2 = 0.0002148.$

$$H_2 = 37.3538 + \frac{(3.5912)^2}{2(32.2)}$$

$$= 37.5541 \text{ ft.}$$

2. Average slope of EGL over Reach 1 is

$$\overline{S}_1 = \frac{S_1 + S_2}{2}.$$

$$\overline{S}_1 = 0.0002097$$

$$H_2' = H_1 + \overline{S}_1 L_1$$

$$= 37.1863 + (0.0002097)(1730)$$

$$= 37.5491 \text{ ft.}$$

G. Check the Guess for h_2

$$|H_2 - H_2'| = 37.5541 - 37.5491 = 0.0050.$$

The guess is close but not quite good enough. Figure 19 gives an exaggerated picture of the location of h_2, H_2, H_2'. Since $H_2' < H_2$ we guess that h_2 was too large. A suggested new guess is to reduce h_2 by $H_2 - H_2'$.

$$\text{New } h_2 = h_2 - (H_2 - H_2')$$

$$= 37.3538 - 0.005$$
$$= 37.3488 \text{ ft.}$$

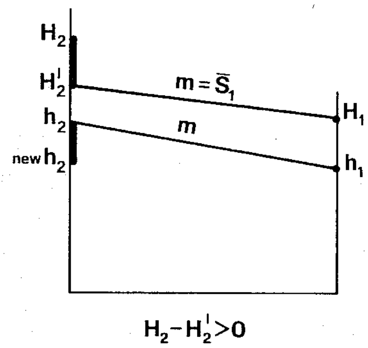


FIGURE 19a

New Guess for h_2

The calculations are:

$$A_2 = 400 + (0.3488)(50) = 417.44$$
$$P_2 = 60 + 2(0.3488) = 60.6976$$
$$V_2 = 1500/417.44 = 3.5933$$
$$R_2 = 417.44/60.6976 = 6.8774$$
$$S_2 = \frac{(0.0002180)(3.5933)^2}{(6.8774)^{4/3}}$$

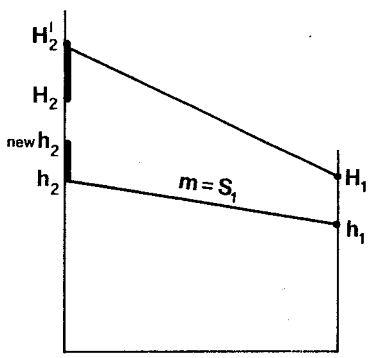
$S_2 = 0.0002152$

$$H_2 = 37.3488 + (3.5933)^2/64.4$$
$$= 37.5493 \text{ ft.}$$

$$\bar{S}_1 = \frac{0.0002152 + 0.0002045}{2}$$
$$= 0.0002099.$$

$$H_2' = 37.1863 + (0.0002099)(1730)$$
$$= 37.5494 \text{ ft.}$$

FIGURE 19b



$H_2 - H_2' < 0$

Check the guess for h_2:
$|H_2 - H_2'| = |37.5493 - 37.5494|$
$= 0.0001 < 0.005.$
This guess is accurate enough.

Comment:

It is possible that in other cases $H_i' > H_i$. This case is also illustrated in Figure 19 b. If this should happen the picture would suggest that the new guess for h needs to be higher than the old one. In either case the formula

$$\textbf{new } h_i = \textbf{old } h_i - (H_i - H_i')$$

will give the desired result. If $H_i' - H_i < 0$, subtracting this quantity will add something to the old h_i. The student should add this formula to the list.

It is possible that a student may carry an extra decimal place, or use fewer places. Since the value of $H_2 - H_2' = 0.005$, small differences may result in $|H_2 - H_2'| < 0.005$. In this case the student can proceed to Section 3 and develop the approximation process for Section 3.

H. Fix a Value for h_2, H_2, and S_2

Since an acceptable guess has been found we can now list acceptable values for h_2, H_2, and S_2. These are logically the guess for h_2 which met the requirements and the corresponding S_2. Whether H_2 or H_2' is used is immaterial since these are within 0.005.

$h_2 = 37.3488$
$H_2 = 37.5493$
$S_2 = 0.0002152.$

Comment:

If the student has found $|H_2 - H_2'| < 0.005$ the choices will be

$h_2 = 37.3538$
$H_2 = 37.5541$
$S_2 = 0.0002148$

I. Find h_3.

The following calculations are the result of repeating the steps in E, F, G, and H.

$h_2 = 37.3488$
$H_2 = 37.5493$
$S_2 = 0.0002152$
$L_2 = 480$
$h_3 = 37.3488 + (0.0002152)(480)$
$= 37.4521$
$\Delta h_3 = 0.4521$
$A_3 = 480 + (45)(0.4521)$
$= 500.3445$
$P_3 = 58 + 2(0.4521)$
$= 58.9042$
$V_3 = 1500/500.3445$
$= 2.9979$
$R_3 = 500.3445/58.9042$
$= 8.4942$

$$S_3 = \frac{(0.0002180)(2.9979)^2}{(8.4942)^{4/3}}$$

$= 0.0001130$

$$H_3 = 37.4521 + \frac{(2.9979)^2}{64.4}$$

$= 37.5917$ ft

$$\overline{S}_2 = \frac{0.0002152 + 0.0001130}{2}$$

$= 0.0001641$

$H_3' = H_2 + \overline{S}_2 \; L_2$

$= 37.5493 + (0.0001641)(480)$
$= 37.6281$ ft
$H_3 - H_3' = -0.0364$

$|H_3 - H_3'| > 0.005$

Since $H_3' > H_3$ the guess for h_3 was too small. Try Again:

new h_3 = old $h_3 + 0.0364$
$h_3 = 37.4521 + 0.0364$
$= 37.4885$

Second calculations for Section 3:

$h_3 = 37.4885$

$\Delta h_3 = 0.4885$

$A_3 = 480 + (45)(0.4885)$
$= 501.9825$

$P_3 = 58 + 2(0.4885)$
$= 58.977$

$V_3 = 1500/501.9825$
$= 2.9882$

$R_3 = 501.9825/58.977$
$= 8.5115$

$$S_3 = \frac{(0.0002180)(2.9882)^2}{(8.5115)^{4/3}}$$
$= 0.0001120$

$$H_3 = 37.4885 + \frac{(2.9882)^2}{64.4}$$
$= 37.6272$ ft.

$$\bar{S}_2 = \frac{0.0002152 + 0.0001120}{2}$$
$= 0.0001636$

$H_3' = 37.5493 + (0.0001636)(480)$
$= 37.6278$ ft.

$H_3 - H_3' = -0.0006$

This is within the acceptable range. We can thus choose the values of h_3, H_3, and S_3:

$h_3 = 37.4885$
$H_3 = 37.6272$
$S_3 = 0.0001120.$

J. Calculate h_{17}

The student should do the remaining calculations using the interactive computer program. If a computer is not available the calculation of h_3 should be considered sufficient provided the student has understood what is going on.

Comment:

Interested students have an opportunity to change various items in the interactive program. In particular they can change the new guess for h_i at any step. One value of the problem is an experience in successive approximations. If the answers for h_{17} agree to two decimal places this should be considered adequate.

VI. The Computer Programs

A computer diskette is available with the AIM packet. Included on the diskette are two solution programs, an interactive program and a direct solution program, written for the Apple II computer. The interactive program is a user friendly program written to let the student work through the problem in a step-by-step procedure allowing the student to make as many decisions as possible. After booting up the AIM diskette, the interactive program can be run by typing RUN HGL INTERACTIVE. The student is now ready to enter data for the first section. The necessary data can be found from the information given for the characteristics of the canal. Once the data has been entered, certain results are calculated and printed for the student to observe. These results include: Hydraulic Radius, R; Velocity, V; the friction slope S_i; and the total energy H_i. It might be of interest to make a table of calculated results while running the program.

For sections 2 through 17, the student again enters the data and is able to guess h_i. A suggested value is given based on the formula in Section V, E. The student can accept this value or make another guess. Using the guess for h_i, the computer calculates and prints the hydraulic radius, R; the Velocity, V; Friction Slopes, S_i and $\bar{S}_i$; and the total energy appoximations, H_i and H_i'. The difference between H_i and H_i' is then printed. If the difference is not small enough, a suggested value for a new h_i is given based on the formula in Section V, G. Again the student is allowed to make a different guess. The calculations are done again and the student decides whether the tolerance between H_i and H_i' is small enough. After accepting the difference, the student can proceed to the next section or exit from the program. After the student exits the program, a table is printed giving the section number, water surface elevation and total energy of the sections calculated. This program is meant to be a tool in the learning process and development of the backwater curve for this problem.

In contrast to the interactive program, the direct solution program is designed to make the desired calculations with as few inputs as possible. The student will be asked to input values for volumetric flow, Q, the roughness factor, n, and the tolerance for the total energies H_i and H_i'. After the student enters these values, the program calculates and prints the results. The characteristics of the canal are given as data in this program and calculations are based on the formulae in the solution section. This program can be run by typing RUN HGL DIRECT after booting up the diskette.

Although the direct solution program is included with the AIM packet, students should be encouraged to write their own programs. To facilitate this, a flow chart and program listing have been included in the appendix. The students can use their imagination in writing their own program or in enhancing the given direct solution program with graphics, etc.

VII. The Written Report

The ability to write is an important skill in any area. Report writing is a regular part of the job of a person working in industry. In the AIM problem the student is playing the role of an employee of Kleinschmidt Inc. This role includes writing a report stating the work requested, the work done, and the results found. In this case the preliminary work could be omitted and the report should deal with parts C to J, the calculation of h_{17} from the data given. Encourage the student to be brief, precise, and clear. The employer wants to know what the answer is, and how the student got that answer, not how hard it was and what false starts were made. Simplifying assumptions should be included. Short but complete sentences are preferred. Spelling and punctuation should be correct. A suggested format is given in the Appendix.

Although students are rarely delighted at the prospect of writing a report they are very proud when they see what they have accomplished! Suggest that they keep it for their portfolio for use when applying for scholarships or jobs.

VIII. Additional Questions and Projects.

A. The Geometry of a Trapezoidal Channel

An open channel is an isosceles trapezoid. (See Figure 20.) The base is 20 feet and the sides make an angle of θ with the base.

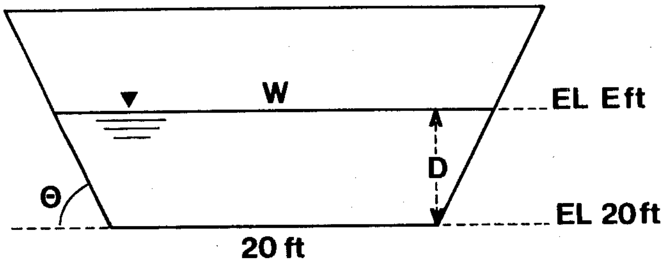


FIGURE 20

1. Suppose $\theta = 45^o$.
   a) Find D, W, A, and P when E = 37 ft.
   b) Calculate R when E = 37 ft.
   c) Find D, W, A, and P when E = 38 ft.
   d) Calculate R when E = 38 ft.
   e) Find D, W, A, and P when E = 39 ft.
   f) Calculate R when E = 39 ft.

2. Create a formula for calculating W, A, P, and R when the depth of the water is D and $\theta = 45^o$. If possible write each quantity in terms of D only. Check your formula using your calculations in question 1.

3. In a channel of the shape in question 1, with $\theta = 45^o$, how does the hydraulic radius change as D increases? Use the formula in question 2 and a selection of values of D to answer this question.

4. Suppose the channel is rectangular, that is, $\theta = 90^o$. Find W, A, P, and R when D = 17. Compare your answer with the answer in question 1 (b). Is R larger or smaller when θ is larger?

5. Suppose $\theta = 30^o$. In this case, $W = 20 + 2\sqrt{3}\,D$ and $P = 20 + 4D$. Calculate W, A, P and R for D = 17. Compare the values of R when $\theta = 90^o$, 45^o, 30^o. How does R change as the channel becomes flatter at fixed depth?

B. The Geometry of a Circular Channel

A circular pipe has radius 10 ft. (See Figure 21.) The depth of the water in the pipe is related to the angle θ shown in the figure.

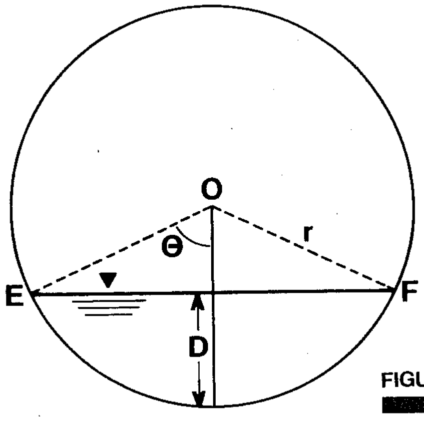


FIGURE 21

1. Suppose the pipe is half full of water. What will the value of θ be in this case? Find A, P, and R in this case.

2. Suppose $\theta = 45^{o}$. Draw a sketch illustrating this case.
   a) What fraction of the circumference of the pipe is P?
   b) Calculate P.
   c) What fraction of the area of the circle of radius 10 is the circular sector OEF?
   d) What is the area of the circular sector OEF? What is the area of the triangle OEF?
   e) Calculate A.
   f) Calculate R.

3. Suppose $\theta = 60^{o}$. Using the steps suggested in question 2, calculate P, A, and R in this case.

4. Suppose $\theta = 120^{o}$. Calculate P, A, and R in this case. Notice that in this case the value of A is greater than the area of the circular sector.

5. (Trigonometry needed). Derive a formula for each of P, A, and R in terms of θ. It will be better to express θ in radians for this formula. Check your formula using the calculations in questions 1, 2, 3, and 4. Will it be necessary to change your formula to adjust to the case when the angle θ is more than $\pi/2$?

6. (Calculus needed.) Find $\lim_{\theta \to 0} R$. Find $\lim_{\theta \to \pi/2} R$. Find $\lim_{\theta \to \pi} R$.

7. (Calculus needed.) Show that if R is to be a maximum or a minimum θ must satisfy the equation

$$\tan 2\theta = 2\theta.$$

8. Consider the equation $\tan x = x$. In our problem $x = 2\theta$, and the values of x are between 0 and 2π. Draw a graph of $y = \tan x$ over the interval $[0,2\pi]$ and also a graph of $y = x$ on the same interval. What is the first point of intersection of these two graphs? In what quadrant is the second point?

9. Use trigonometric tables, or your calculator, to approximate the value of θ which makes R a maximum. You might wish to calculate R for values of θ beginning with $2\pi/3$ radians, and increasing by about 0.1 radians each time.

C. Examine the Backwater Curve

1. Draw a bar graph showing h_{17} at each of the 17 sections. In order to make the change in h_{17} more visible, you might draw your bars beginning at elevation 36 or 37 ft above mean sea level. Do you think the slope of the water surface would be visible to a person looking at the canal?

2. Assume that the HGL is approximated by a set of straight lines joining the calculated heights at the sections. Find the slope of each of these lines. In which reach is the slope greatest? In which reach is it least? Study the data given in the table to see if there might be a physical reason for your answers.

3. The friction slope is not printed automatically at the end of either program. You can ask the program for a listing of S(I) by giving the command:

```
FOR I = 1 TO 17:  PRINT I, S(I):  NEXT
```

   Make a listing of S_i for each section i.

4. From the listing in Question 3, calculate the average friction slope for each reach. Compare these values with the slopes calculated in Question 2. Are the HGL and the EGL "nearly parallel"?

D. What if . . .

One of the most important uses of the computer in industry is in computer simulation. This technique makes it possible to predict the results of some procedures without investing the time and money needed to actually carry them out.

1. Would it be desirable to do further work on the canal bed in order to make it smoother? If the roughness factor were reduced to 0.015, what would be the value of h_{17}?

2. Suppose no work is done to make the canal smoother. What would h_{17} be if the roughness factor n = 0.035?

3. What is the result of changing the volumetric flow? If Q = 1000 cu ft/sec, what is h_{17}? If Q = 1250 cu ft/sec what is h_{17}? What if Q = 1750 cu ft/sec? What if Q=2000 cu ft/sec?

4. The upstream end of the canal opens out of the Connecticut River. The body of water in the river near the opening is called the "Pond". In order to maintain the volumetric flow, the water surface elevation in the Pond must be at least 0.5 ft above the surface elevation at the upstream end of the canal. Suppose the surface elevation of the pond is 40.2 ft. What is the largest possible volumetric flow through the canal system?

5. In the study requested in this problem the engineers require $|H_i - H_i'| < 0.005$ at each section. If this requirement is changed, what change in h_{17} and H_{17} will result? Make a

   table showing the values of h_{17} and H_{17} using

   the requirement $|H_i - H_i'| < T$ for

   T = 0.05, 0.01, 0.005, 0.001, 0.0005, 0.0001. Which requirement seems to you most reasonable from a practical point of view?

answers:

A. 1(a) D=17; W=54; A= $\frac{17}{2}$ (54 + 20) = 629;

P = 20 + 34$\sqrt{2}$ = 68.1

1(b) R = 9.24

1(c) D=18; W=56; A=684; P=70.9

1(d) R = 9.65

1(e) D=19; W=58; A=741; P=73.7

1(f) R = 10.05

2. W=20 + 2D; A=D(20 + D); P=20 + 2D$\sqrt{2}$

$$R = \frac{D(20 + D)}{20 + 2D\sqrt{2}}$$

3. R increases as D increases.

4. W=20; A=340; P=54; R=6.30. R is smaller.

5. W=78.9; A=840.6; P=88; R=9.55.
For fixed D, R increases as θ decreases.

B. 1. $\theta = 90^\circ$; $A = \frac{\pi r^2}{2} = 50\pi$;

$P = \frac{2\pi r}{2} = 10\pi$; R = 5. Notice that in this case R = diameter/4.

2. a) 1/4
b) P = 5π = 15.7
c) 1/4
d) Area of circular Sector = 25π = 78.54
Area of triangle = 50
e) A = $25\pi - 50$ = 28.54
f) R = $(25\pi - 50)/5\pi = 5 - \frac{10}{\pi}$ = 1.82

3. $P = \frac{2\pi r}{3} = 20.94$

$A = \frac{\pi r^2}{3} - \frac{r^2}{4}\sqrt{3} = 61.42$

R = 2.93

4. $P = \frac{2}{3}(2\pi r) = 41.89$

$A = \frac{2\pi r^2}{3} + \frac{r^2\sqrt{3}}{4} = 209.44$

R = 5

5. $P = \frac{\theta}{\pi}(2\pi r) = 2r\theta$, θ in radians

Sector area $= r^2\theta$
Triangle area
$= 2(1/2)(r \sin\theta)(r \cos\theta)$

$= r^2 \sin\theta \cos\theta$

$A = r^2(\theta - \sin\theta\cos\theta)$

$$R = \frac{A}{P} = \frac{r^2(\theta - \sin\theta\cos\theta)}{2r\theta}$$

$$= \frac{r}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right)$$

No change is needed since $\cos\theta < 0$ when $\pi/2 < \theta < \pi$.

6. Since $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$, $\lim_{\theta\to 0} R = \frac{r}{2}(1-1) = 0$

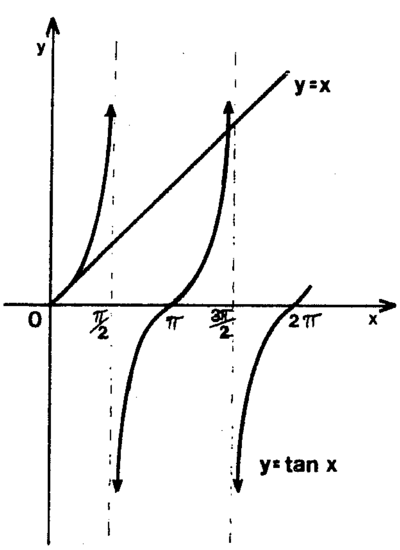
$\lim_{\theta\to\pi/2} R = r/2$

$\lim_{\theta\to\pi} R = r/2$

7. $$\frac{dR}{d\theta} = \frac{r(\sin 2\theta - 2\theta\cos 2\theta)}{4\theta^2}$$

$\frac{dR}{d\theta} = 0$ if $2\theta\cos 2\theta - \sin 2\theta = 0$.

This implies $\tan 2\theta = 2\theta$ if $\cos 2\theta \neq 0$.



8. $\tan x = x$ if $x = 0$, and also for

$\pi < x < \frac{3\pi}{2}$. If $x = 0$, $\theta = 0$ and

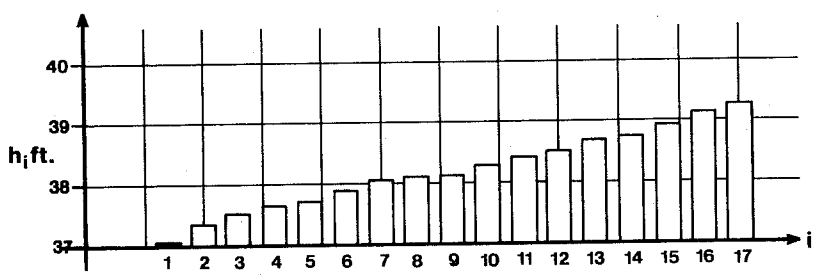
R is not defined but $R \to 0$ as $\theta \to 0$.
R is increasing when $\tan x < x$ and R is decreasing when $\tan x > x$. Maximum R occurs at $\tan 2\theta = 2\theta$.

9. θ approximately 2.245 radians $\cong 129^\circ$.
Maximum R is

$$5\left(1 - \frac{\sin(4.49)}{4.49}\right) = 6.09 \text{ ft.}$$

C. 2 and 4.

Reach	HGL Slope	Average Friction Slope
1	0.0002045	0.0002097
2	0.0002906	0.0001634
3	0.0001248	0.0001184
4	0.000125	0.0001273
5	0.0001576	0.0001349
6	0.0001703	0.0000821
7	0.0000270	0.0000345
8	0.0000452	0.0000575
9	0.0000640	0.0000599
10	0.0000615	0.0000725
11	0.0001070	0.0001005
12	0.0000956	0.0000955
13	0.0000853	0.0000797
14	0.0000742	0.0000754
15	0.0000767	0.0000760
16	0.0000752	0.0000733



Reach 2 has greatest HGL slope. Reach 7 has smallest HGL Slope. I have no definitive answer for why. A good guess might be that Reach 2 is narrowest and short, while Reach 7 is the longest and widest.

D. 1. $h_{17} = 38.3057$

2. $h_{17} = 40.9590$

3. $Q = 1000$, $h_{17} = 38.2296$
$Q = 1250$, $h_{17} = 38.7302$
$Q = 1750$, $h_{17} = 39.7603$
$Q = 2000$, $h_{17} = 40.2687$

4. If $Q = 1720$ $h_{17} = 39.6986$
If $Q = 1721$, $h_{17} = 39.7006$
Choose the smaller, $Q = 1720$.

5.

T	h_{17}	H_{17}
0.05	39.1727	39.2549
0.01	39.2392	39.3202
0.005	39.2448	39.3257
0.001	39.2416	39.3225
0.0005	39.2404	39.3214
0.0001	39.2404	39.3214

IX. References:

An encylopedia is the best source from which students with little background can get an understanding of the language and concepts used in this problem. Here the terms will be explained without assuming a technical background. Students should be encouraged to do some general reading about water power and other sources of energy. They should be discouraged if they wish to hunt for a place where a similar problem is done. Try to let them feel the excitement of creating a solution for themselves.

Technical information can be found in Fluid Mechanics and Hydraulics by Ranald V. Giles, Schaum's Outline Series in Engineering, McGraw-Hill, New York (1962). This includes background material but little relating directly to the problem. A recent moderately technical book which is also quite readable is Fluid Mechanics with Engineering Applications, by Robert L. Daugherty, Joseph B. Franzini, and John E. Finnemore, published by McGraw-Hill, New York (1985).

The Handbook of Hydraulics mentioned by Dr. Kleinschmidt has gone through many editions. The most recent is 6th edition by Brater and King, McGraw-Hill, 1982. Earlier editions give more insight into the problems (for example the 4th edition, 1954). Also suggested are Wastewater Engineering, Collection and pumping of waste-water, Metcalf and Eddy, Inc., edited by G. Tchobanoglous, McGraw-Hill, 1981; Hydraulics, 5th Edition, King, Wisler, and Woodburn, John Wiley and Sons, Inc., 1956; Practical Hydraulics, Simon, John Wiley and Sons, Inc., 1976.

Since books in hydraulic engineering are not usually on the shelves of public libraries, you may have some difficulty in finding these unless you are close to a University with an Engineering College. Remember that it is not necessary to have a detailed understanding of the background material. The interest of the problem lies in devising and using a mathematical technique of successive approximations.

X. Appendix

A. Notation

A	section area
EGL	energy grade line
g	acceleration due to gravity (32.2 ft/sec^2)
H	total energy head
h	potential energy head
HGL	hydraulic grade line
n	roughness factor (0.022)
P	wetted perimeter
Q	volume flow rate (1500 cu ft/sec)
R	hydraulic radius
S	friction slope
$\overline{S}_i$	average friction slope over Reach i
V	velocity

B. Formulas

$$R = A/P$$

$$H = h + \frac{V^2}{2g}$$

$$V = \frac{Q}{A}$$

$$V = \frac{1.49}{n} R^{2/3} S^{1/2}$$

$$S = \frac{n^2 V^2}{(1.49)^2 R^{4/3}}$$

$$A_i = A_{0i} + W_i \Delta h_i$$

$$P_i = P_{0i} + 2\Delta h_i$$

$$\overline{S}_i \approx \frac{S_{i+1} + S_i}{2}$$

C. Glossary

Average friction slope - the average of the friction slopes at the points of a reach

Backwater Curve - Hydraulic Grade line

Energy Grade Line - Plot of the total energy head $H = h + V^2/2g$

Friction Slope - Slope of the Energy Grade Line

Hydraulic Grade Line - Plot of the potential energy head, h

Hydraulic radius - ratio of cross-sectional area to wetted perimeter

Mean sea level - an average level of ocean waters

Reach - a theoretical subdivision of the canal

Reach length - the distance between sections of the canal

Roughness factor - a constant reflecting the physical characteristics of the canal

Section area - the area of a cross-sectional slice of the canal

Water surface width - the distance from one bank of the canal to the other along the water surface

Wetted perimeter - the length of contact between the water and a solid boundary

D. Report Format

OPENING MATERIAL

Title Page	
Table of Contents	Section headings and page numbers.
Glossary	This should include only terms special to this problem.
List of symbols	
Abstract	A brief statement of what the student was asked to do and the answer. (No more than four or five lines.)

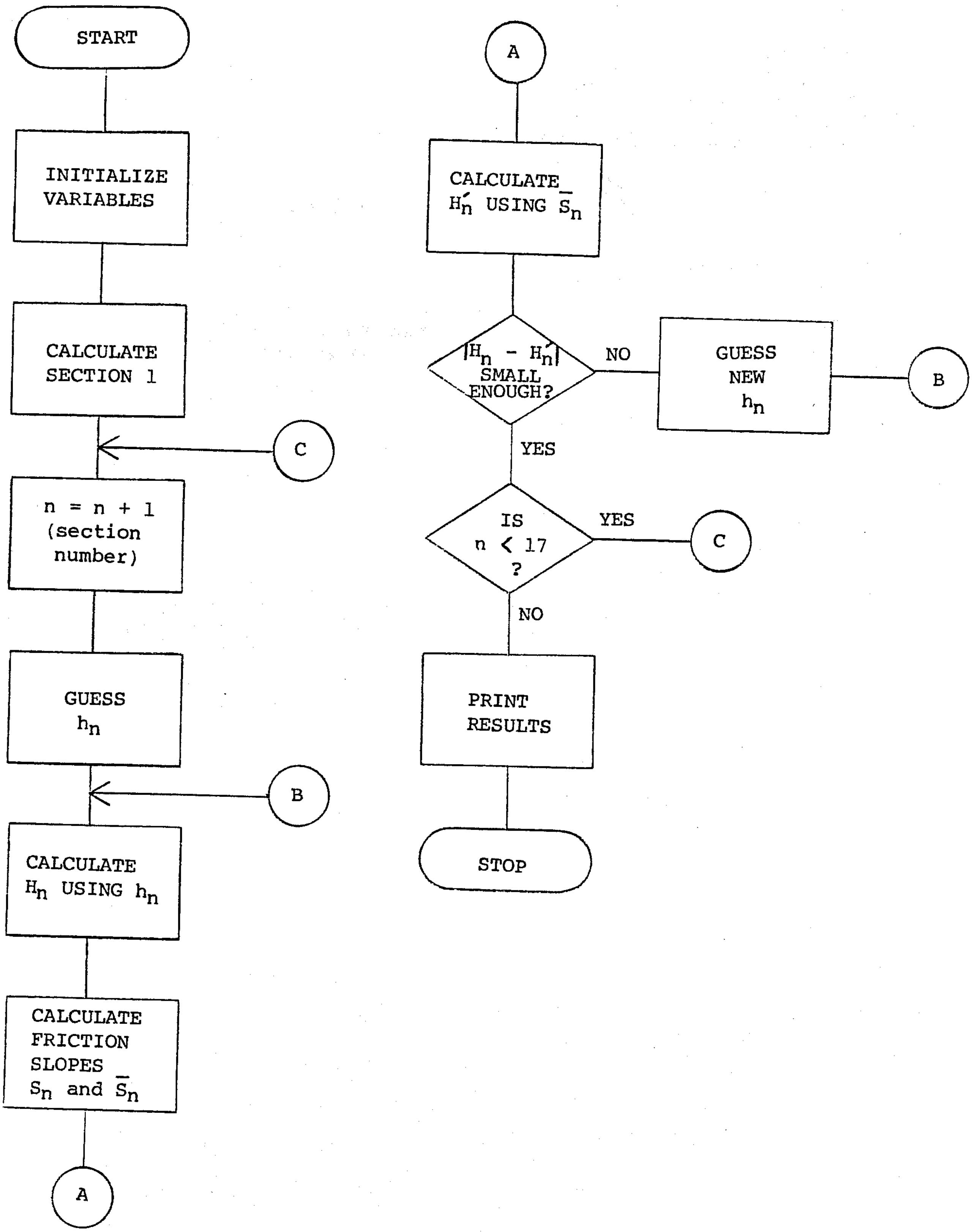
BODY

Introduction	Statement of what was assigned and why it was needed.
Discussion	The simplifying assumptions. An outline of the formulas given and any formulas that were derived, and an outline of the procedure.
Results	A more detailed statement of the results than that given in the abstract.
Conclusions and Recommendations	If any value judgment can be given it should be included here. For example, is the canal a good source of power?

APPENDIX

References and Acknowledgements	Any books or articles consulted. Any people who helped.
Computation	A flow chart or program if that was needed. Detailed results or calculations.

E. Flow Chart and Program



```
]LIST

100  REM        BACKWATER CURVE PROBLEM
110  REM           A DIRECT SOLUTION
120  REM
130  REM        WRITTEN BY JEFF DIMICK
140  REM             OCTOBER 1985
150  REM
160  REM        VARIABLE DECLARATION:
170  REM
180  REM   A = CROSS SECTIONAL AREA
190  REM   P = WETTED PERIMETER
200  REM   W = WATER SURFACE WIDTH
210  REM   L = REACH LENGTH
220  REM   M = SECTION NUMBER
230  REM   R = HYDRAULIC RADIUS
240  REM   S = FRICTION SLOPE
250  REM   H = WATER SURFACE ELEVATION (HGL)
260  REM   H1 = TOTAL ENERGY (EGL - USING GUESS FOR HGL)
270  REM   H2 = TOTAL ENERGY (EGL - USING AVG. S)
280  REM   V = VELOCITY
290  REM   Q = VOLUMETRIC FLOW
300  REM   N = ROUGHNESS FACTOR
310  REM   G = GRAVITY
320  REM   D = DIFFERENCE USED TO CALCULATE ADJUSTED DATA
330  REM   T = TOLERANCE BETWEEN THE EGL'S
340  REM
350  DIM A(2,17),P(2,17),S(17),W(17),H1(17),H2(17),R(17),L(17),H(17)
360 G = 32.2: REM   GRAVITY
370  REM
380  DATA  433,64,60,1730,400,60,50,480,480,58,45,1320,480,68,58,400,480,
     72,63,1160,500,102,99,750,900,102,100,2730,700,92,87,990,600,94,91,2
     170
390  DATA  650,86,81,2700,500,84,80,900,490,99,95,1925,500,84,80,760,523,
     83,78,2225,500,79,74,1720,500,78,70,2140,500,78,70,0
400  REM
410  REM   READ DATA
420  FOR M = 1 TO 17
430  READ A(1,M),P(1,M),W(M),L(M)
440  NEXT
450  REM
460  REM   PRINT INSTRUCTIONS AND COMMENTS
470  HOME
480  HTAB (7): INVERSE : PRINT "A BACKWATER CURVE PROBLEM": NORMAL : PRINT

490  PRINT "THIS PROGRAM GIVES A DIRECT SOLUTION": PRINT : PRINT "TO THE
     BACKWATER CURVE PROBLEM": PRINT : PRINT "DESCRIBED IN THE STUDENT RE
     SOURCE BOOK": PRINT
500  PRINT "OF THE AIM PACKET.  THE DATA USED IN": PRINT : PRINT "THIS PR
     OGRAM CAN BE FOUND IN THE TABLE": PRINT : PRINT "IN SECTION V.B.  TH
     E FORMULAE USED": PRINT : PRINT "ALSO COME FROM SECTION V.  YOU WILL
      BE": PRINT
510  PRINT "ASKED TO ENTER VALUES FOR:": PRINT : PRINT  TAB( 9);"1)  VOLU
     METRIC FLOW ": PRINT  TAB( 9);"2)  ROUGHNESS FACTOR": PRINT  TAB( 9)
     ;"3)  TOLERANCE ": PRINT
520  PRINT : PRINT  TAB( 6);"TYPE <RETURN> TO CONTINUE";: GET A$
530  HOME : PRINT "THE CALCULATED WATER SURFACE ELEVATION": PRINT : PRINT
     "AND THE TOTAL ENERGY WILL BE PRINTED ": PRINT : PRINT "WHEN THE PRO
     GRAM IS FINISHED RUNNING.": PRINT : PRINT
540  REM
```

```
550  REM   INPUT Q,N, AND T
560  PRINT  TAB( 6);"VOLUMETRIC FLOW ";: INPUT Q
570  PRINT : PRINT  TAB( 6);"ROUGHNESS FACTOR ";: INPUT N
580  PRINT : PRINT  TAB( 6);"TOLERANCE FOR THE EGL'S ";: INPUT T
590  HOME : VTAB (10): PRINT  TAB( 6): PRINT "CALCULATING, PLEASE WAIT!"
600  REM
610  REM   CALCULATIONS FOR SECTION 1
620  GOSUB 880
630  REM
640  REM   CALCULATIONS FOR SECTIONS 2-17
650  FOR M = 2 TO 17
660 FLAG = 0: REM   FIRST TIME FOR SECTION
670  REM
680  REM   CALCULATE H1(M)
690  GOSUB 970
700  REM
710  REM   CALCULATE H2(M)
720  GOSUB 1080
730  REM
740  REM   CHECK DIFFERENCE OF H1 AND H2
750  IF  ABS (H2(M) - H1(M)) < T THEN 800
760  REM
770  REM   CALCULATE NEW H(M)
780 H(M) = H(M) - (H1(M) - H2(M))
790 FLAG = 1: GOTO 680
800  NEXT
810  REM   CALCULATIONS COMPLETE - PRINT RESULTS
820  HOME : HTAB (16): PRINT "RESULTS": PRINT
830  PRINT "  SECTION  WATER SURFACE  TOTAL ENERGY ": PRINT "   NUMBER EL
     EVATION (HGL)     (EGL)    ": PRINT
840  FOR M = 1 TO 17
850  PRINT  TAB( 6);M; TAB( 14);H(M); TAB( 28);H1(M)
860  NEXT
870  END
880  REM
890  REM   SECTION 1 CALCULATIONS
900 R(1) = A(1,1) / P(1,1)
910 V = Q / A(1,1)
920 S(1) = (V * N) ^ 2 / (1.49 ^ 2 * R(1) ^ (4 / 3))
930 H(1) = 37.0
940 H1(1) = H(1) + V ^ 2 / (2 * G)
950  REM
960  RETURN
970  REM
980  REM   SECTIONS 2-17 CALCULATION OF EGL -- H1(M)
990  IF FLAG = 0 THEN H(M) = H(M - 1) + S(M - 1) * L(M - 1)
1000 D = H(M) - 37.0
1010 A(2,M) = A(1,M) + W(M) * D
1020 P(2,M) = P(1,M) + 2 * D
1030 R(M) = A(2,M) / P(2,M)
1040 V = Q / A(2,M)
1050 H1(M) = H(M) + V ^ 2 / (2 * G)
1060  REM
1070  RETURN
1080  REM   SECTIONS 2-17 CALCULATION OF EGL -- H2(M)
1090 S(M) = (V * N) ^ 2 / (1.49 ^ 2 * R(M) ^ (4 / 3))
1100 SAVG = (S(M - 1) + S(M)) / 2
1110 H2(M) = H1(M - 1) + SAVG * L(M - 1)
1120  REM
1130  RETURN
```