# A Backwater Curve for the Windsor Locks Canal

# Student Resource Book


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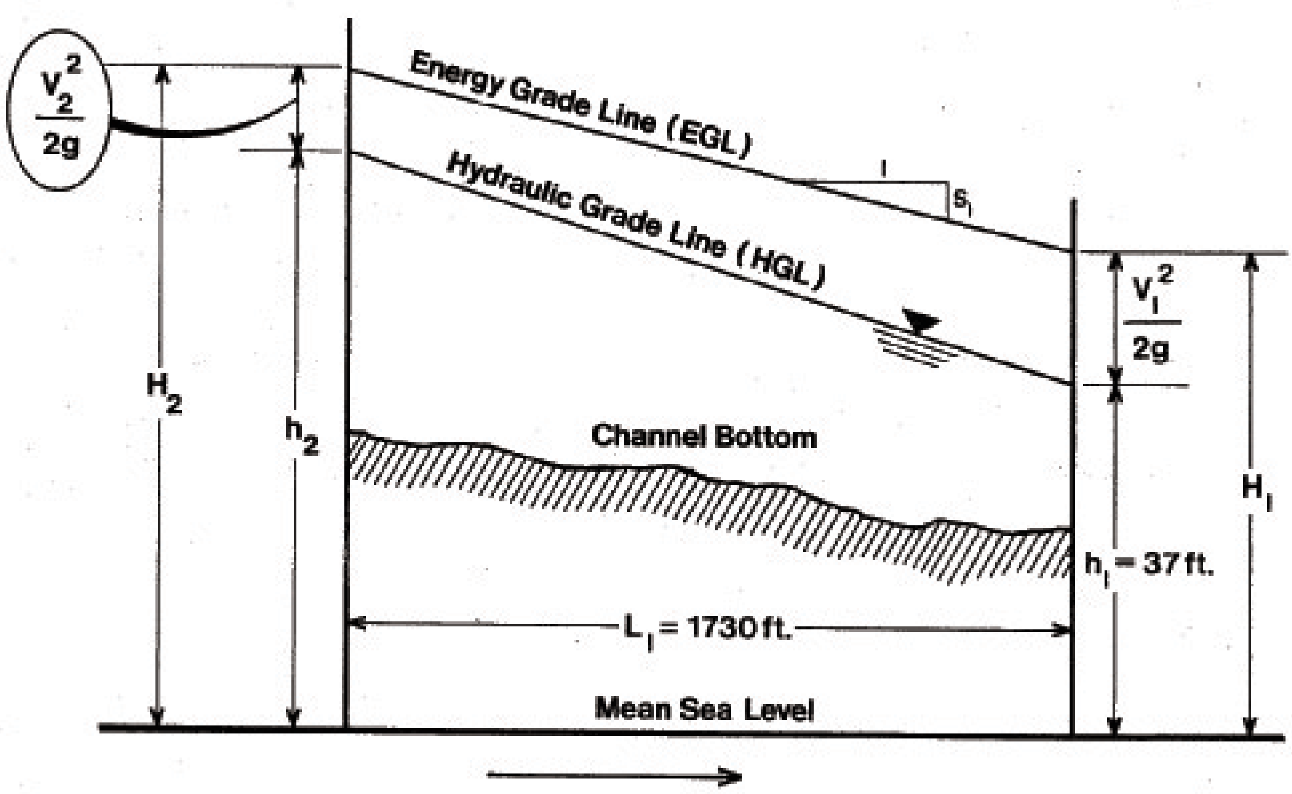

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# I. Introduction

When you turn on the lights where does the electricity come from? You get it from whatever power company produces and distributes electricity in your area. How do the power companies produce electricity? Large generators in power stations produce almost all our electric power.

Turbines drive these electric generators. A turbine is just a wheel turned by the force of moving fluid such as water or steam. The turbine does not create power, it changes the force of moving fluids into rotary motion which is used to turn a generator to produce electricity.

There are two principal types of turbines, steam turbines and water turbines. The steam turbines are driven by steam produced either by a coal or oil-burning furnace or by a nuclear reactor. Water turbines are turned by flowing water from a dam or waterfall.

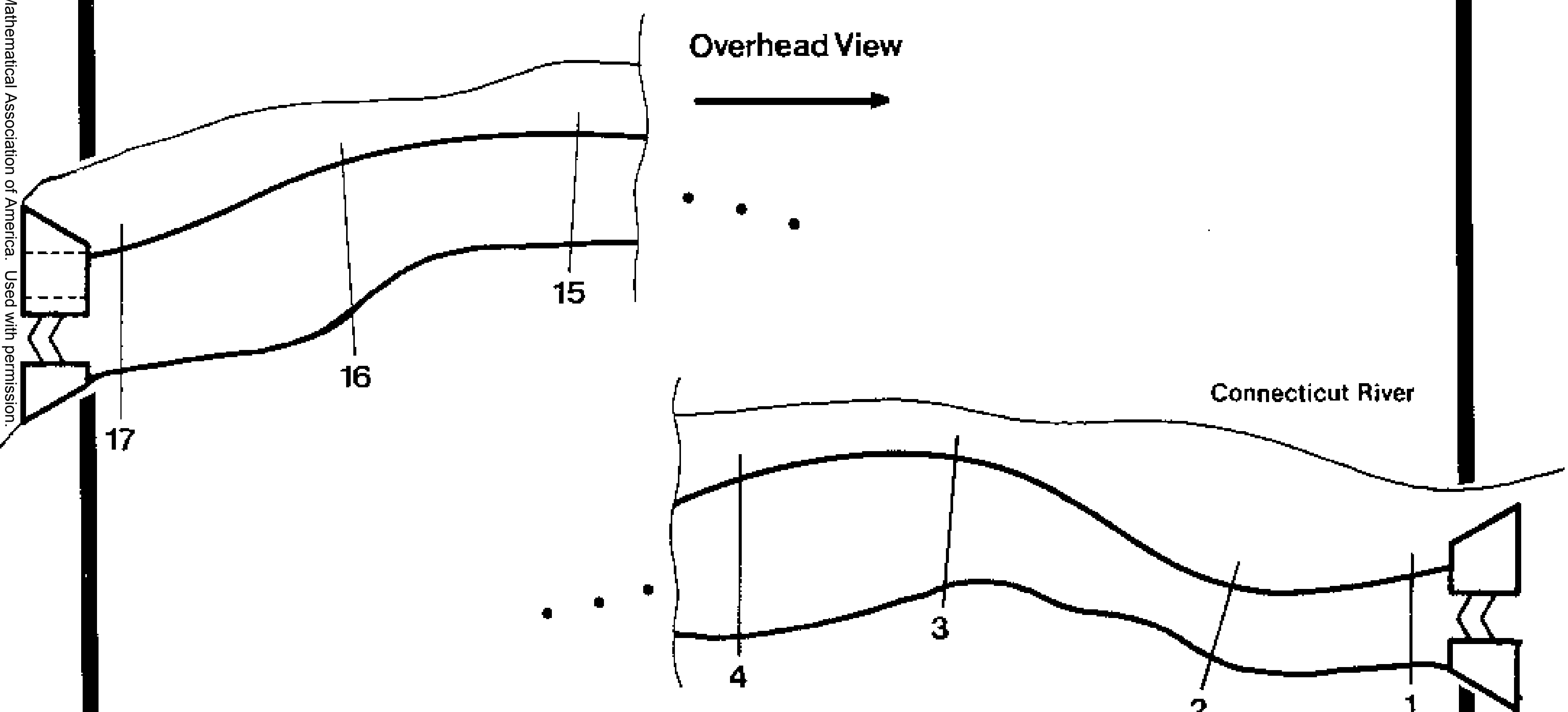
The power of a water turbine depends on (1) the volume of flowing water, and (2) the distance that the water falls before it strikes the turbine wheel. Any encyclopedia will give you more detailed information about different kinds of turbines and generators and how these work. These details are not required for the present study. Our AIM is to study the Windsor Locks Canal and decide whether it is a possible source of water power.

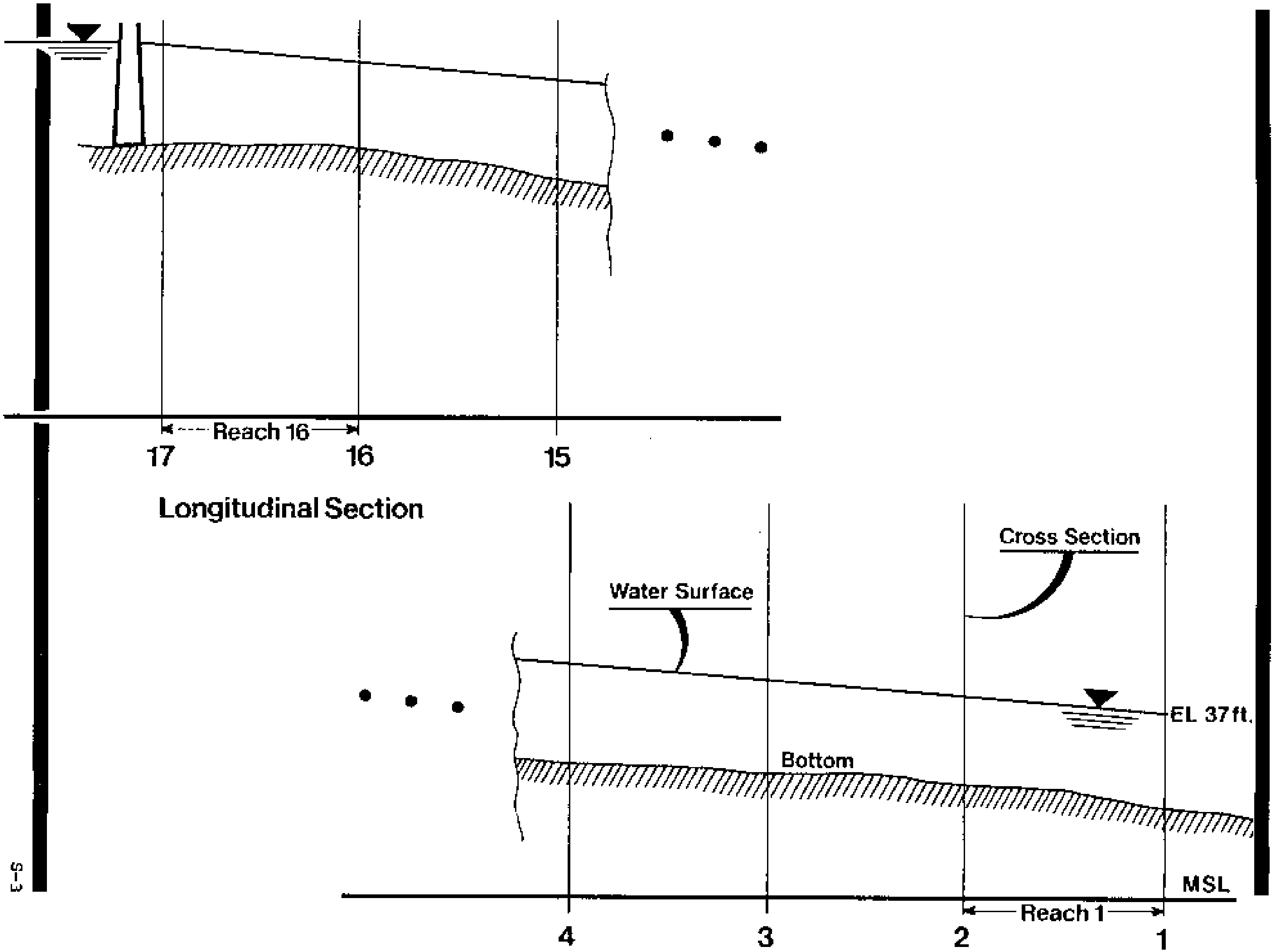
# II. Description of the Canal

Windsor Locks Canal is an old navigation canal parallel to the Connecticut River and bypassing a 5-mile stretch of the river where there are rapids. In these five miles the river drops about 30 ft. At each end of the canal is a dam, and at the downstream end is a water turbine. The project under consideration is to use the canal for the production of electricity. This may involve some dredging and reshaping of the canal, removing sludge from the bottom and any large rocks. It will be necessary to bring water down the canal to the turbine at the rate of 1500 cu ft/sec. An outlet pipe at the downstream

FIGURE 1

## Plan of Windsor Locks Canal






Longitudinal Section

end carries the water to the turbine. In order to achieve maximum power a water control device fixes the water surface elevation downstream at 37 ft above mean sea level. We do not want the water to drop much during its run down the canal since we want as large a change in height as possible at the turbine.

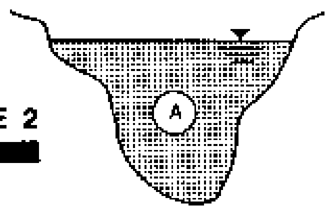
All elevations are given as a distance above mean sea level. Sea level is the level of ocean waters. However, this is not the same in all parts of the world. Also sea level changes with the tides. For purposes of measurement a definite average has been agreed on by scientists who use the idea of sea level in their studies. This average is called the mean sea level.

Figure I shows a plan of the Windsor Locks Canal. Since the canal is between four and five miles long, the characteristics of the flow are not the same throughout the length of the canal. In the course of their preliminary survey the engineers have decided to think of the canal as subdivided into sixteen parts in each of which the flow can be assumed to be nearly uniform. Each of these parts is called a reach. Since the characteristics of the flow change from one reach to the next, the engineers measure the canal at the points where one reach ends and the next begins. They consider an imaginary cross section of the canal taken perpendicular to the direction of flow of the water. They then measure the elevation at several horizontal distances across the canal bed. These measurements are used to calculate the quantities described in the following paragraphs. Because there are 16 reaches there are 17 measured cross sections. Section 1 is at the downstream end and section 17 at the beginning of the canal. The Reach Length, L ft, is the distance between sections.

The hydraulic data needed at each section are: cross-sectional area, wetted perimeter and water surface width.

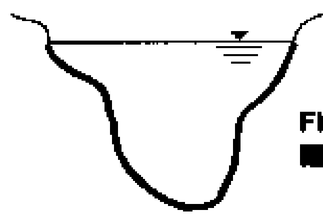
The Section Area, A, is actually the area of a cross-sectional slice of the canal. It is the number of square feet enclosed by the following boundaries: the canal bottom, the canal sides, and the water surface. (Figure 2)

FIGURE 2




Section Area, A sq ft

FIGURE 3

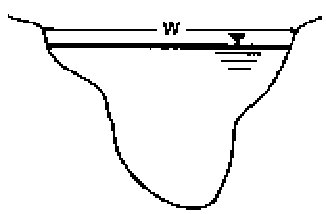


Wetted Perimeter, P ft

The Wetted Perimeter, P, is that portion of the perimeter of the cross section where there is contact between the water and a solid boundary. (Figure 3)

The Water Surface Width, W, is the distance from one bank of the canal to the other along the surface of the water at a given section. (Figure 4)

FIGURE 4




Water Surface Width, W ft

The Section Area and the Wetted Perimeter are used to calculate the hydraulic radius which gives a measure of flow properties.

The Hydraulic Radius, R, is defined as the ratio of the cross-sectional area to the wetted perimeter at a section.

$$R = A/P \text{ ft.}$$

The physical characteristics of the canal are described by a constant called Manning's Constant. If the surface is very rough (much vegetation on the canal bottom, rocks, etc.) this constant will be large. This constant is thought of as a roughness factor. The Roughness Factor, n, is assumed to be constant for the Windsor Locks Canal and to have the value $n = 0.022$.

# III. Open Channel Flow

The flow of water in an open channel can be thought of in two ways. We can think in terms of the quantity of water that passes across a particular section in a certain length of time. We measure volume in cubic feet and the length of time chosen is one second. The letter Q is used to represent this quantity. The volume flow rate is then Q cu ft/sec.

It is also important to know the speed at which the water is moving. The velocity tells how many feet a particular particle of water moves in one second. The letter V is used to designate this quantity. The velocity at any point is V ft/sec.

The amount of water that crosses a given area in one second is the product of area and velocity. This gives the equation

$$Q \text{ cu ft/sec} = A \text{ sq ft} \times V \text{ ft/sec}$$

or

$$V = \frac{Q}{A} \text{ ft/sec.}$$

The engineers plan to keep the volume flow rate constant at 1500 cu ft/sec. Uniform flow implies the velocity is the same at every point in the fluid. This is certainly not the case in the canal as a whole, since the area of a cross section is not constant. However there will be stretches of the canal in which the flow is approximately uniform. This is taken into consideration by the engineers in choosing the sixteen reaches of the canal.

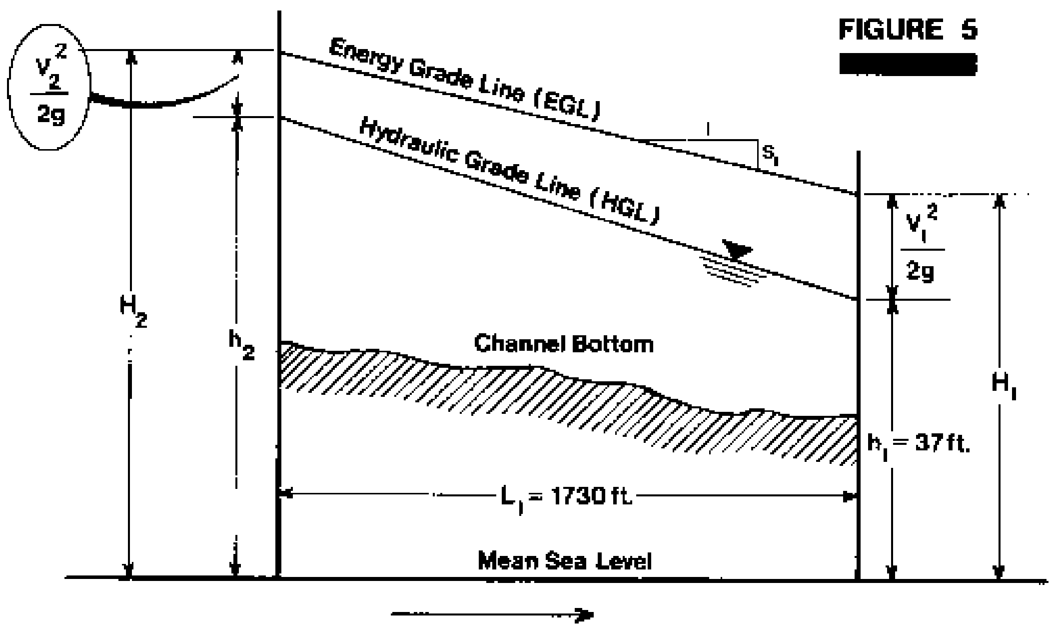
There are two forms of energy present in fluid flow, potential energy and kinetic energy. The potential energy of a particle of liquid depends upon its elevation above an arbitrarily chosen reference plane. In our case the reference plane is mean sea level. A fluid particle of weight W pounds a distance h ft above mean sea level, has a potential energy of Wh foot-pounds. Its potential energy per unit of weight is Wh/W = h ft. This value is called the potential energy head.

Kinetic energy comes from motion. For velocity V ft/sec the kinetic energy per unit weight, kinetic energy head, is $\frac{V^2}{2g}$ ft where g is the acceleration due to gravity (32.2 ft/sec$^2$). The total energy head is

$$H = h + \frac{V^2}{2g} \text{ ft.}$$

Since h and V are different at different locations in the canal H also depends on position. The plot of H along the length of the canal is called the Energy Grade Line (EGL). (See Figure 5.)

FIGURE 5




The potential energy head, h, is also the elevation of the surface of the water. The plot of h along the length of the canal is called the Hydraulic Grade Line (HGL). Because $\frac{V^2}{2g}$ is always positive h is less than H and the EGL is above the HGL.

The water surface elevation at Section 1, near the turbine is kept at 37 ft. The HGL is called the Backwater Curve because it is a plot of the water surface elevation upstream from a position where the surface elevation is held constant.

The slope of a straight line is the change in the vertical position over the change in the horizontal position, that is, rise/run. In a rectangular coordinate system, if $(x_1,y_1)$ and $(x_2,y_2)$ are points on a line the slope is given by

$\frac{y_2 - y_1}{x_2 - x_1}$. The EGL is usually not a straight line. The slope at any point on this curve is the slope of the tangent line at that point.

For an ideal canal of constant cross section and with uniform roughness, the EGL, the HGL, and the bottom of the canal are all parallel. For the Windsor Locks Canal these curves are not quite parallel even in a single reach. The slope of the EGL, denoted by S, is the number of feet per foot that the total energy head drops. The reason for the drop in EGL is the energy loss due to friction. The slope, S, is given the name Friction Slope. An important empirical formula based on experiment connects velocity, hydraulic radius, roughness factor, and friction slope:

$$V = \frac{1.49}{n} \sqrt[3]{R^2} \sqrt{S}.$$

(In our case n = 0.022.)

Fractional exponents are sometimes more convenient than the radical notation. Recall that

$$\sqrt[3]{R} = R^{1/3} \text{ so } \sqrt[3]{R^2} = R^{2/3}.$$

Also $\sqrt{S} = S^{1/2}$. Thus

$$V = \frac{1.49}{n} R^{2/3} S^{1/2}.$$

Since V and R are different at different sections, the friction slope at one end of a reach is different from the friction slope at the other end. Although it changes gradually along the reach, it is sometimes useful to think of an average friction slope over the reach. We use $\overline{S}_i$ to represent the average slope over the ith reach. We cannot calculate it exactly but we can approximate it by the average of $S_i$ and $S_{i+1}$. The average slope over Reach 3, for example, would be approximately

$$\overline{S}_3 \approx \frac{S_4 + S_3}{2}.$$

Figure 5 illustrates these concepts for the first reach. The fundamental principle is the principle of energy balance. The total energy at Section 2 is equal to the total energy at Section 1 plus the loss due to friction. Near Section 1 that loss is $S_1$ ft/ft.; near section 2 it is $S_2$ ft/ft.

# IV. The Problem in General

Kleinschmidt Associates is undertaking an analysis and potential revision of the Windsor Locks Canal. The principal thrust of the analysis is to answer a question frequently asked in searching for sources of water power — What is the capacity of a given canal with the upstream and downsteam water levels given?

In particular, the question to be answered is the following: **What upstream water surface elevation would be necessary to deliver 1,500 cubic feet of water per second to the downstream end of the Windsor Locks Canal?** The water surface elevation of the downstream end is 37 feet above mean sea level.

The analysis required in determining the answer to this question is described in section V, the Problem in Detail. It is this analysis that Kleinschmidt Associates is asking you to perform.

# V. The Problem in Detail

## A. Preliminary Activities

1. The following table shows measurements of the existing canal bed at various positions across Section 2.

| Horizontal Position (ft) | Elevation (ft) |
|---|---|
| 0.0 | 41.0 |
| 2.5 | 33.5 |
| 9.0 | 31.0 |
| 11.0 | 31.0 |
| 15.0 | 30.0 |
| 18.0 | 29.5 |
| 21.0 | 30.0 |
| 31.0 | 30.0 |
| 44.0 | 33.0 |
| 51.5 | 34.0 |
| 51.5 | 42.0 |

Plot these on graph paper. Use a scale no smaller than 1 inch = 10 ft. Sketch the existing canal bed. Draw a horizontal line showing the water level at an elevation, h, of 37 ft above mean sea level.

2. The engineers plan to dredge the canal, reshape certain sections, remove sludge from the bottom, etc. The revised section has the following measurements.

| Horizontal Position (ft) | Elevation (ft) |
|---|---|
| 0.0 | 41.0 |
| 2.5 | 33.5 |
| 22.0 | 25.5 |
| 31.0 | 25.5 |
| 51.5 | 34.0 |
| 51.5 | 42.0 |

On the graph drawn in 1, plot these points and join them by straight lines to obtain a sketch of the revised section. Calculate from your graph:

The area of cross section of the water, A
The wetted perimeter at Section 2, P
The width of the water surface, W

In paragraphs $A_2$ and $A_3$ round all calculations to one decimal place. Based on these calculations, calculate the Hydraulic Radius, R, when the water is at elevation 37 ft above mean sea level.

3. Suppose the water level increases to 39 ft above mean sea level. Draw the 39 ft water surface on your graph. For the revised Section 2 find A, P, W, and calculate R when h = 39 ft.

4. In the calculation in 3, assume that the increase in the area of cross section is approximated by a rectangle, that is, that the sides of the canal are vertical instead of their actual shape. Calculate A, P, W, and R for water level at 39 ft under this assumption. How much difference does it make to assume that the increase in area is a rectangle?

## B. The Given Data

The following table shows the dimensions of the canal as the engineers plan to revise it. Since the canal is divided into 16 reaches, there are 17 sections. Section 1 is at the point where 1500 cu ft of water is delivered through a large pipe to the turbine in the power station. The water surface elevation just before the turbine is fixed at 37 ft above mean sea level.

Column 1 of the table gives the number of the section, beginning with section 1 at the downstream end of the canal, just before the turbine. The second column gives the cross-sectional area at each section assuming that the

TABLE 1

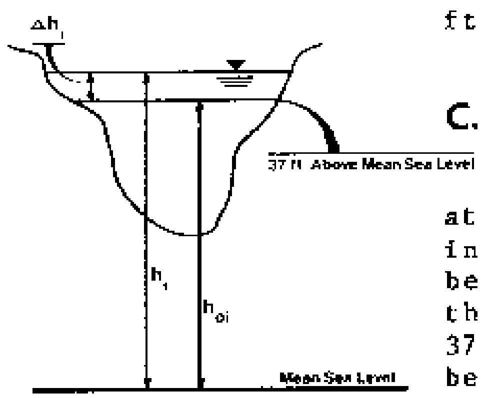
WINDSOR LOCKS CANAL HYDRAULIC DATA
For Water Surface Elevation 37 ft

| Section | Section Area | Wetted Perimeter | Water Surface Width | Reach Length |
|---|---|---|---|---|
| $i$ | $A_{0i}$ sq ft | $P_{0i}$ ft | $W_i$ ft | $L_i$ ft |
| 1 | 433 | 64 | 60 | 1730 |
| 2 | 400 | 60 | 50 | 480 |
| 3 | 480 | 58 | 45 | 1320 |
| 4 | 480 | 68 | 58 | 400 |
| 5 | 480 | 72 | 63 | 1160 |
| 6 | 500 | 102 | 99 | 750 |
| 7 | 900 | 102 | 100 | 2730 |
| 8 | 700 | 92 | 87 | 990 |
| 9 | 600 | 94 | 91 | 2170 |
| 10 | 650 | 86 | 81 | 2700 |
| 11 | 500 | 84 | 80 | 900 |
| 12 | 490 | 99 | 95 | 1925 |
| 13 | 500 | 84 | 80 | 760 |
| 14 | 523 | 83 | 78 | 2225 |
| 15 | 500 | 79 | 74 | 1720 |
| 16 | 500 | 78 | 70 | 2140 |
| 17 | 500 | 78 | 70 | 0 |

surface of the water is at 37 ft above mean sea level. The third column gives the wetted perimeter for each section, and the fourth column gives the water surface width. The fifth column gives the reach length. The first reach extends from section 1 to section 2 and has length 1730 ft. The second reach extends from section 2 to section 3 and has length 480 ft, and so on. The entry at the end of this column indicates that section 17 is the upstream end of the canal.

Notice that the notation for section area in the table is $A_{0i}$, read "A sub zero i". The subscript 0 emphasizes the fact that these measurements all assume that the water surface is 37 ft above mean sea level. The suscript i indicates which section is being considered. Since there are 17 sections i can be any integer from 1 to 17 as indicated in column 1. For example, $A_{06}$ = 500 sq ft.

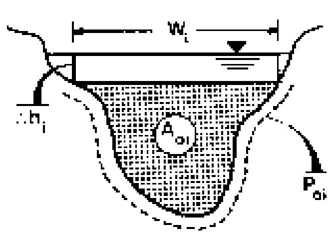
Similarly, $P_{0i}$ means the wetted perimeter of Section i when the water surface elevation is 37 ft.




**Change in h at Section i**

## C. Adjust the Data

The water surface at section 1 is maintained at 37 ft above mean sea level. The flow of water in the canal, as well as the slope of the canal bed, causes the elevation of the water surface at the up-stream sections to be slightly higher than 37 ft. The values in the table will thus need to be adjusted for the increased elevation before the other constants are calculated. For simplicity assume that the increase in the section area is shaped like a rectangle. A good mathematical notation for the increase in h over 37 ft is Δh, so that at section i the new water surface elevation, $h_i$, is 37 + $\Delta h_i$.




**Additional Water as a Rectangle**

1. Assume that the increase of area of section i is in the form of a rectangle of height $\Delta h_i$. Develop a formula for $A_i$ and $P_i$ in terms of $A_{0i}$, $P_{0i}$, and $\Delta h_i$.

2. Check your formula with the work done in A problem 4. What is Δh in this problem?

## D. Calculate the Data for Section 1

In this and all following sections assume the data in the table is exact. Calculate each S to seven decimal places, and all other quantities to four decimal places. The measurements of Section 1 are given in the first row of the table.

1. Calculate $V_1$, $R_1$, and $S_1$ assuming that Q = 1500 cu ft/sec.

2. Calculate $H_1$. (Use $g = 32.2$ ft/sec$^2$.)

## E. The Data for Section 2

The data given in the table for Section 2 assume that the water surface is at an elevation of 37 ft. To adjust the data to the proper water surface level at section 2 the value of $h_2$ is needed.

1. Think of a way to guess an appropriate value for $h_2$. The friction slope at Section 1 might be a help in this.

2. Using your guess, calculate $A_2$ and $P_2$.

## F. Calculate the Data for Section 2

On the basis of your guessed value of $h_2$ you can now calculate $H_2$ in two ways.

1. Using your guess for $h_2$ and the corresponding estimates of $A_2$ and $P_2$, calculate $V_2$, $R_2$, $S_2$, and $H_2$.

2. If you knew the average slope of the energy grade line you could calculate $H_2$ from $H_1$. Estimate the average slope, $\overline{S}_1$, of the EGL for Reach 1. Use this to calculate $H_2$. Call this value $H_2'$.

3. Compare the value of $H_2$ found in 1 with the value $H_2'$ found in 2.

## G. Check Your Guess for $h_2$

In this study we require that $H_i$ and $H_i'$ differ by less than 0.005.

1. If $|H_2 - H_2'| < 0.005$ go on to calculations for Section 3.

2. If $|H_2 - H_2'| \geq 0.005$ think of a way to improve your guess for $h_2$.

## H. Fix a Value for $h_2$, $H_2$, and $S_2$

Your improved guess may be good enough to satisfy the condition in G. Or it may need more improvement. Find a value of $h_2$ that is acceptable.

1. Using your new guess for $h_2$, calculate new values for $A_2$, $P_2$, $V_2$, $R_2$, $S_2$, and $H_2$. Calculate $H_2$ using the average slope obtained from $S_1$ and the new $S_2$. Again check your guess by the method of G.

2. If needed, repeat the process until an $h_2$ is found that makes $|H_2 - H_2'| < 0.005$.

Assign $h_2$ and $S_2$ the values used in your acceptable guess. Choose either $H_2$ or $H_2'$ in this guess for the assigned value of $H_2$.

## I. Find $h_3$.

Now that the data for Section 2 is found, the steps in E, F, G, and H can be repeated to find $h_3$.

## J. Find $h_{17}$

Continue this process step by step until $h_{17}$ is found. You may use the interactive computer program to make these calculations. Although $h_{17}$ is the most important part of your result, it would be helpful if you include also a table showing $h_i$ and $H_i$ at each section.

The actual work requested in this problem is the calculation of $h_{17}$. However, you may be wondering how your answer relates to the question that prompted the study. Is the canal a viable source of water power?

The answer to this question depends on the relation between the $h_{17}$ which you calculate and the water level in the Connecticut River at the upstream end of the canal. The body of water in the river near the opening of the canal is called the "Pond." In order to maintain the required volumetric flow, the level of the pond must be far enough above $h_{17}$ to move the water down the canal and to allow for the small losses we have disregarded in the investigation, for example, evaporation and run off. "Far enough" in this case is 1/2 ft. or more.

We are not given the level of the river at the opening to the canal. In fact, this level would fluctuate during the year. So our answer would have to be stated in the form: "The canal is a viable source of water power provided the level in the pond is ..."