# Pricing Auto Insurance

# Teacher Resource Book

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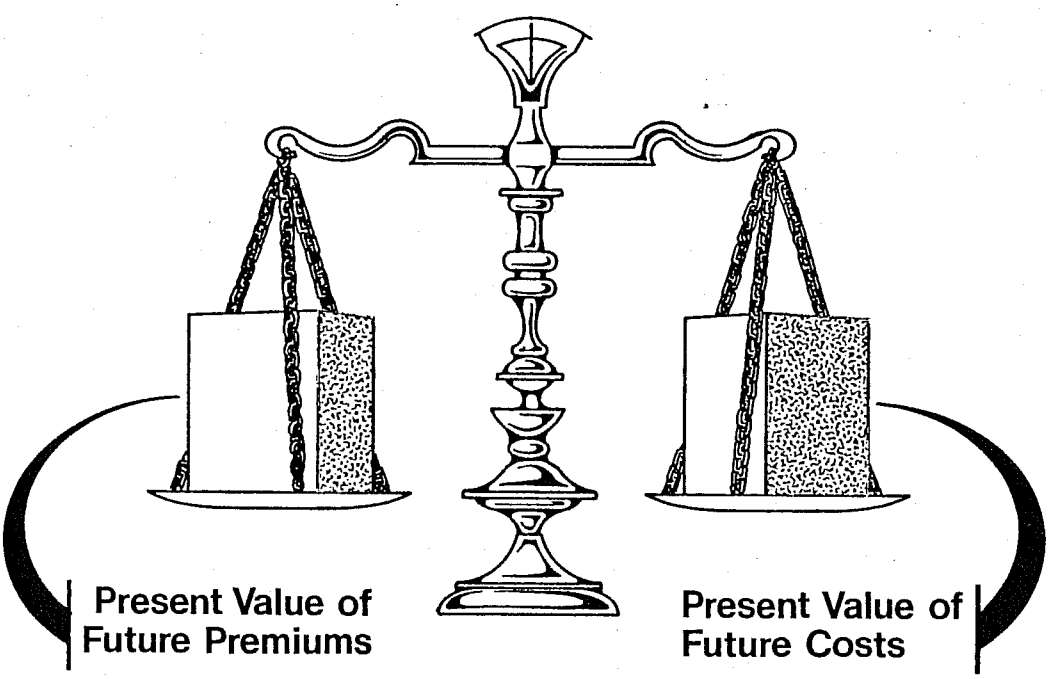
Fundamental Equation of Value



This book is one part of a complete learning module for the problem "Pricing Auto Insurance." The entire learning module consists of three video cassettes entitled "The Problem", "Problem Preparation", and "A Solution"; a student resource book; a teacher resource book; and a microcomputer diskette for use on an Apple computer.

The book was prepared, published, and distributed by the Mathematical Association of America under the National Science Foundation Grant number MDR-8470469, entitled Applications in Mathematics (AIM).

The materials produced under the AIM project are based on industry-related applied mathematics problems. They have been designed and produced to offer high school teachers a strategy for providing their students an experience in using their reading, writing, and mathematical abilities in solving real problems. In this way the students may realize that these skills are inter-related and that the mastery of them is of vital importance in their future career opportunities.

For more information about this and other AIM modules, write to:

AIM: The Mathematical Association of America
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Washington, D.C. 20036

**Title:** Pricing Auto Insurance

**Source:** Alfred L. Litchenburg, F.S.A., M.A.A.A.
Vice-President and Chief Actuary
American Fidelity Companies
Oklahoma City, OK

**Prerequisites:** Current enrollment in Algebra I.

**Skills Needed:** Arithmetic, percent, solution of a linear algebraic equation in one variable.

**Summary:** An analysis is made of receipts and expenses incurred in providing auto insurance. On the basis of empirical data (provided) the students are to calculate the fair price of auto insurance for specified age groups. The concept of present value is required in the problem. A preliminary problem omitting this is included. Use of the computer is optional.

**Comments:** Necessary background information is provided. The students are encouraged to find out more about types of insurance. The interest of the problem lies in the use of simple mathematics to analyze a business situation which is likely to be of importance to them.

**Suggested Classroom Uses:**

A unit in a mathematics course (such as Business Math, Algebra I, or Algebra II)

Independent study

A project for a math club

Enrichment in Algebra I (Background material parts A through D; Preliminary Problem; Additional Questions)

Enrichment in Algebra II (Background material parts E through H; The Problem; Additional Questions)

Enrichment in Business Math

Enrichment in Pre-Calculus Math (Geometric Series)

Experience in Computer Simulation (Teacher Resource Book, Section IX)

Pricing Auto Insurance

# Teacher Resource Book

## Table of Contents

# I. Introduction

This book is the Teacher Resource Book for the industry-related problem, "Pricing Auto Insurance". It is one part of a coordinated package of materials called an AIM Learning Module. The total AIM module consists of: Video I, "The Problem"; Video II, "Problem Preparation"; the Student Resource Book; the Teacher Resource Book; one microcomputer BASIC program; and Video III, "A Solution".

All three Videos are on-site conversations with the industrial representative who has actually worked with this problem. In Video I the discussion centers on the problem and what it means to the company as a whole. In Video II the industrial representative discusses some background material and provides illustrations and hints to assist the student in making progress toward a solution. The discussion in this video also gives the student information about a career as an actuary. In Video III the industrial representative provides his solution of the problem.

Three versions of the problem are included in the Student Resource Book. In the preliminary problem, the mathematics required is arithmetic, percent, and solving a linear equation in one unknown. In the problem which is discussed in Video I, the idea of present value is introduced as a refinement of the preliminary problem. The use of present value involves no additional skills but does introduce an unfamiliar concept. The third problem, called the challenge problem, simply asks for the solution of the same problem using letters in place of numbers. Such a solution would enable the student to write a computer program. An interactive computer program makes it possible to change many of the constants in the problem and thus answer some interesting "What if" questions.

The interest of the problem lies in the way that simple mathematical skills are used to make important decisions in the business world. The subject of auto insurance is one with which high school students will soon be involved.

Section II of the Student Resource Book gives a discussion of the various elements involved in pricing insurance. This is set up in eight rather brief sections each of which contains suggested questions designed to test the student's understanding of the material read. Answers to the questions in the Student Resource Book, Section II, are given in the Teacher Resource Book, Section IV.

Section III of the Student Resource Book states the problem in general. Section IV sets out the relevant data regarding frequency of accidents in various age groups. In Section V the preliminary problem, the problem, and the challenge problem are stated in detail, along with the necessary data about costs.

Section III of the Teacher Resource Book, entitled "Where to Begin", discusses ways in which the teacher might introduce this particular problem and points out some activities which might help the students get started. Section IV contains some comments on the background material provided, and complete solutions to the questions asked in the background discussion.

Section V, entitled "Developmental Approach", sets forth the thought processes through which a student might proceed in reaching a solution to the problem. Since one way of formalizing your thinking is to ask yourself a series of questions, the developmental approach is written completely in question format. This section can be used by the teacher as a source of leading questions to guide student discussion of the problem in a class. Or it can be given to the students to help them in thinking. If the problem is to be used as independent study, this section will be especially useful. It is probable that for the student's first experience with an AIM module the developmental approach will be important. After the student has worked through one or two of the AIM modules, the student will have acquired enough problem-solving technique to make the developmental approach unnecessary.

Section VI, A Solution in Detail, gives a complete solution of each of the three problems and makes some comments on questions which might arise.

Section VII discusses the computer program provided and explains ways in which the initial information can be altered to answer questions which might arise. This program, or one like it developed by the student, will be very useful in getting a feel for how the pricing of auto insurance is affected by social and economic factors, for example: increase in accident rate, rising prices, rising or falling interest rates, gender of the insured, age of the insured, etc.

In Section VIII the writing of a report is discussed.

In Section IX a collection of related questions is provided. Some of these are extra practice in using interest and percent. Some are "What if" questions as described above. Some are an extension of the ideas of interest and present value to repayment of loans.

# II. Teaching Strategies

There are many exciting ways in which the AIM materials can be used in the high school curriculum. You are, of course, free to use your creativity to modify these and to devise others that fit your individual situation.

Whatever method you choose, the goal is twofold: to have the students experience mathematics in an industrial setting; to raise the student's awareness of careers in mathematics.

## A. A Unit in a Mathematics Course

The objective is to involve the student in a discovery-learning approach geared to developing and sharpening the following skills:

1) reasoning and model building

2) real-world problem solving

3) communicating verbally about mathematics

4) writing technical material concisely and accurately

5) making use of resource materials

6) using the computer in problem solving.

Step 1. Preparation. Give each student a copy of the Student Resource Book. (Permission is granted to copy these for classroom use.) The student reads it and becomes acquainted with the problem and its setting.

Step 2. Video I. The students view Video I. Through this on-site video they meet the industrial representative who discusses the problem and some related material.

Step 3. Getting Started. The students study the problem, become familiar with the terms and consult reference books as needed. They work through the background sections A through D, and begin to develop a mathematical model that provides the context in which the problem can be solved. They talk about the problem, its setting, the technical terms, the assumptions made, and ways in which they might attack the problem. They try their skill by working the preliminary problem. At this point they may feel the need for reinforcement and extra direction. They now view Video II. The examples worked by the industrial representative will be meaningful for them, and his further hints will help them with the problem.

Step 4. Creating a Solution. The students work as a class or as individuals and discuss their work. At this stage you, as teacher, skillfully nudge the students toward a solution. To help you accomplish this, Section V of this book, entitled "Developmental Approach", provides a succession of questions some of which you might use to stimulate discussion. A complete solution of the problem is provided in Section VI. However, the teacher's role is not to provide a solution but rather to encourage and tease the students to find their own solution. Students interact with their peers and their instructor and also use the resources of their campus (library, computer center, faculty, etc.). Students come to class ready to report on their mathematical progress. When solutions are obtained, they present their solutions and field questions on their work.

Step 5. The Computation. A solution program is provided on a diskette for use with Apple II.

The program is user-friendly and allows the student to enter a variety of initial data. This program makes it possible for the student to answer "What if" questions which are likely to arise. Some such questions are suggested in Section IX. Students will think of many on their own. Some may wish to write their own solution program. The challenge problem helps them develop the general structure they will need in order to do this.

Step 6. The Report. Each student writes a technical report on the problem and its solution. The report is discussed in Section VIII and a suggested format is given in Section XI.

Step 7. Video III. The students view Video III, "A Solution". At this time they can compare their solution with that provided on the video.

## B. A Unit for Independent Study

The method described in A is an "ideal" way to use the AIM materials in a class-oriented problem-solving situation. The same general method is equally effective when used as individual instruction or independent study for one or more students. In such a case class discussion gives way to periodic teacher conferences. If the student is short of time or unable to proceed, the teacher can provide the list of questions given in the developmental approach (Section V) to lead the thought processes of the student.

## C. Enrichment in a Variety of Courses

Use AIM Videos I, II, and III as a lecture presentation to a class when you wish to stimulate interest in mathematics by demonstrating an application of the material they are studying.

Assign parts of the problem when the class work deals with some skill used in the problem solution. In this case you might show AIM Video I to acquaint the class with the problem setting and then give a brief discussion of the method of solution.

Students can experience computer simulation by using the computer solution with different data. The "What if" problems suggested in Section IX illustrate the use of the computer to answer questions about the effect on insurance rates of economic and other changes.

## D. Project for a Math Club

The AIM materials make an exciting series of programs for a math club. The technique described in Section A can easily be adapted to this setting. The fact that math clubs include students at various levels adds interest in the sharing of skills.

## E. Developing Career Awareness

All three videos give a firsthand picture of an actuary at work. Implicitly all three raise the student's consciousness of actuarial work as a career and of the importance of mathematics in his/her future career choices. What this work involves is discussed briefly in Video II.

## F. Group Presentations

Use the AIM Video as the basis of a presentation at a regional meeting of NCTM.

Use AIM Videos and written materials as a resource for a workshop for your area group of mathematics teachers.

# III. Where to Begin

A major hurdle in problem solving is deciding where to begin. Some background reading will certainly be helpful. Students may wish to discuss auto insurance with their parents. They should find out the insurance laws in their state.

The first formal step is to read through the Student Resource Book. At this first reading the students need not answer the questions in Section II, but should try to get the general idea that the key issue is to balance receipts and costs when the pricing of insurance is undertaken. They should then view Video I, The Problem.

Before any attempt is made to work the problems in Section V, the students should study Section II in detail. Since some words may be unfamiliar it would be wise for them to make a glossary as they proceed. A sample glossary is included in Section XI. Some of the questions in

Section II may seem very simple as much of the material here is a review of arithmetic. Students should not skip the simpler questions since they lead into the solution of the problems in Section V.

Notice that the preliminary problem in Section V requires only the material in parts A through D of Section II. It might be helful for the students to work the preliminary problem at this point. A class in Algebra I might wish to do only this part of the module. An Algebra II class should master the entire section. If, when Section II D is completed, the students feel that they need help or reinforcement, this would be a good time to view Video II, Problem Preparation. They should then do the preliminary problem.

Some discussion in class of the relationship between the concept of interest and the problem at hand might be helpful. Students should appreciate the fact that the insurance company is being very fair in its estimates of cost and of receipts by expressing everything in terms of present value at the beginning of the year. Perhaps the students would like to guess how the premiums calculated in the problem compare to those calculated in the preliminary problem. Will they be larger, smaller, or the same?

# IV. Background Comments, Answers to Tops

The background provided in Section II includes material related to insurance and also some mathematical material, such as percent, interest, present value, and the geometric series. Although the mathematics may be familiar to many students, it is included here since many high schools do not teach courses in business mathematics. In order to be sure the students get a working understanding of the background, some short problems called **TOPs** (Try Out Problems) are inserted periodically. Even students who feel very sure of themselves should work these problems. For your convenience solutions are included here.

**TOP 1.**
Let P = annual premium
total premiums = total claim costs
$1000\,P = (50)(\$5000)$

$$P = \frac{(50)(5000)}{1000}$$

$$= \$250.$$

**TOP 2.**

(a) Average claim cost $= \frac{\$250,000}{50}$

$= \$5000.$

(b) Let P = annual premium

(i) $1000\ P = \$250,000$

$P = \frac{250,000}{1000}$

$= \$250.$

(ii) Males:

$500\ P = (30)(\$5000)$

$P = \frac{(30)(5000)}{500}$

$= \$300.$

(iii) Females:

$500\ P = (20)(\$5000)$

$P = \frac{(20)(5000)}{500}$

$= \$200.$

**TOP 3.**

Average claim cost $= \frac{\$250,000}{50}$

$= \$5000.$

Let P = annual premium

Males under 30: $100\ P = (10)(\$5000)$

$P = \frac{(10)(5000)}{100}$

$= \$500.$

Males 30 and over: $400\ P = (20)(\$5000)$

$P = \frac{(20)(5000)}{400}$

$= \$250.$

Females under 30: $100\ P = (5)(\$5000)$

$P = \frac{(5)(5000)}{100}$

$= \$250.$

Females 30 and over: $400\ P = (15)(\$5000)$

$P = \frac{(15)(5000)}{400}$

$= \$187.50.$

**TOP 4.**
Let P = monthly premium

$$(1000)(12\ P) = (50)(\$5000) + (1000)(\$10)$$

$$P = \frac{250{,}000 + 10{,}000}{(12)(1000)}$$

$$= \$21.67.$$

**TOP 5.** (a) 0.65
(b) 0.01
(c) $0.45 \times 60 = 27$
(d) $0.07 \times A = 0.07\ A$.

**TOP 6.**
Let P = monthly premium

$$\text{total premiums} = \text{claim costs} + \text{expenses} + \text{risk charge}$$

$$(1000)(12\ P) = (50)(\$5000) + (1000)(\$50) + (0.05)(1000)(12\ P)$$

$$(1000)(12\ P) - (0.05)(1000)(12\ P) = (50)(5000) + (1000)(50)$$

$$(0.95)(1000)(12\ P) = 250{,}000 + 50{,}000$$

$$P = \frac{250{,}000 + 50{,}000}{(0.95)(1000)(12)}$$

$$= \$26.32.$$

**TOP 7.**
(a) Balance after 2 compounding periods is $A(1 + I)^2$.
For the third period:
interest earned $= I(A)(1 + I)^2$
new balance $= A(1 + I)^2 + IA(1 + I)^2$
$= A(1 + I)^2\ (1 + I)$
$= A(1 + I)^3$.
(b) $A(1 + I)^4$.

**TOP 8.**
(a) $I = (0.09)(1/12)$
$= 0.0075$.
(b) Eight months of interest will have been earned.
$B = A\ (1 + I)^n$, $A = \$50$, $I = 0.0075$, $n = 8$
$B = (\$50)(1.0075)^8$
$= \$53.08$.
(c) Seven months of interest will have been earned.
$A = \$50$, $I = 0.0075$, $n = 7$
$B = (\$50)(1.0075)^7$
$= \$52.68$.

(d) Six months of interest will have been earned.

$A = \$50,\ I = 0.0075,\ n = 6$

$B = (\$50)(1.0075)^6$

$= \$52.29.$

(e) Let S equal the current balance of your bank account.

January 1 $S = \$50$

February 1 $S = \$50 + \$50(1.0075)$

March 1 $S = \$50 + \$50(1.0075) + \$50(1.0075)^2$

.

.

.

September 1 $S = \$50 + \$50(1.0075) + \ldots + \$50(1.0075)^8$

$= \$463.74.$

**TOP 9.**

$$A = \frac{B}{(1 + I)^n}, \quad B = \$100,\ I = 0.01,\ n=12$$

$$A = \frac{\$100}{(1.01)^{12}}$$

$$= \$88.74.$$

**TOP 10.**

(a)

$$PV = \$1 + \frac{\$1}{1.01} + \frac{\$1}{(1.01)^2} + \frac{\$1}{(1.01)^3} + \ldots + \frac{\$1}{(1.01)^{11}}$$

$$= \$11.36.$$

(b)

$$PV = \$50 + \frac{\$50}{1.01} + \frac{\$50}{(1.01)^2} + \frac{\$50}{(1.01)^3} + \ldots + \frac{\$50}{(1.01)^{11}}$$

$$= \$568.08.$$

(c)

$$PV = \$C + \frac{\$C}{1+I} + \frac{\$C}{(1+I)^2} + \frac{\$C}{(1+I)^3} + \ldots + \frac{\$C}{(1+I)^{11}}$$

$$= \$C\left[1 + \frac{1}{1+I} + \frac{1}{(1+I)^2} + \ldots + \frac{1}{(1+I)^{11}}\right]$$

or

$$PV = \$C\,(1 + v + v^2 + \ldots + v^{11}) \text{ where } v = \frac{1}{1 + I}.$$

**TOP 11.**

(a) $S_n = \frac{a(1 - r^n)}{1 - r}$

$n = 12,\ a = \$1,\ r = \frac{1}{1.01} = 0.990,$

$S_{12} = \frac{\$1\ (1 - 0.990^{12})}{1 - 0.990}$

$= \$11.36.$

(b) $n = 12,\ a = \$50,\ r = \frac{1}{1.01} = 0.990,$

$S_{12} = \frac{\$50(1 - 0.990^{12})}{1 - 0.990}$

$= \$568.08.$

(c) $n = 12,\ a = \$C,\ r = \frac{1}{1 + I}$ ,

$S_{12} = \frac{\$C\ [1 - (\frac{1}{1 + I})^{12}]}{1 - \frac{1}{1 + I}}$ .

# V. Developmental Approach

## A. Preliminary Problem

1. What am I looking for in this problem?

   What does "premium" mean?
   Is it part of income or expense for the company?
   How frequently is the premium paid?
   How shall I designate this unknown?

2. Think about the preliminary problem, part (a).

   How many policies shall I consider?
   How many premiums are received in a year?
   How shall I write the total premiums?
   How many claims are there?
   How much is paid out to settle the claims?
   Are there any other costs connected with claims?
   What is the cost of claim processing?
   What other costs does the company have?
   What is the cost of commissions?
   What is the cost of issuing the policies?
   What is the cost of premium billing?
   What is the risk charge?

3. Now what quantities do I balance to find the monthly premium?

   What are the total receipts?
   What is the total cost?
   What is the equation?
   What is the unknown?
   How can I solve for P?

4. Think about the preliminary problem, part (b).

   How does part (b) differ from part (a)?
   Which entries must I change?
   Now what?

   "Piece of Cake!"

## B. The Problem

1. What does investment earnings mean?

   Is this a part of costs or receipts?
   Which side of the equation does it affect?
   Why are 12 terms needed in finding present value of premiums?
   Is PV(Premiums) more or less than (12,000)(P)?
   What is PV(Claims) in part (a)?
   Is this more or less than (132)(5000)?
   What is PV(Cost of issuing policy)?
   What is PV(Cost of premium billing)?
   What is PV(Cost of claim processing)?
   What is PV(Risk charge)?

2. Now how do I find P in part (a)?

   What is the fundamental equation?
   How does it look when I put in the values I have found?
   Will P be larger or smaller than it was in the preliminary problem, part (a)? Let's calculate and see.

3. Consider the problem, part (b).

   How is part (b) different from part (a)?
   Which entries must I change?
   What is P this time?
   Compared with the preliminary problem is my answer reasonable?

## C. Challenge Problem

1. What thought processes are required?
   What is the only difference between the problem and the challenge problem?
   Do I still need to use the Fundamental Equation of Value?
   Why don't I need to know which group I am considering?

2. What is the "answer" to the challenge problem?
   Is it a number?
   Is it a formula?
   It might be fun to write a computer program.

# VI. A Solution in Detail

## A. Preliminary Problem

Let $P be the monthly premium to be charged. Assume 1000 policies are issued. The general procedure is to calculate both sides of the balance equation, set up the equation, and solve for P.

Total Income = Total Costs

Step 1. There are 12 premiums in a year and 1000 policies, so that the total amount received in premiums is (12)(1000) P = 12,000 P. This is the same for parts (a) and (b).

Step 2. The three major categories on the right hand side are: claim payments, expenses, and risk charge.

| | Part (a) | Part (b) |
|---|---|---|
| Claim Payments | | |
| Number of accidents | 132 | 72 |
| Average claim | $5,000 | $5,000 |
| Total Claim Payments | $660,000 | $360,000 |
| Expenses | | |
| Commissions, taxes, fees = 15% of premium = (0.15)(12000) P | $1,800 P | $1,800 P |
| Cost of issuing policy = $20 per policy | $20,000 | $20,000 |
| Cost of billing and management = $12 per policy | $12,000 | $12,000 |
| Cost of claim processing = $10 per claim | $1,320 | $720 |
| Total Expenses | $1,800 P + 33,320 | $1,800 P + 32,720 |
| Risk Charge | | |
| 5% of premiums = (0.05)(12000 P) | $600 P | $600 P |
| TOTAL COSTS | $2,400 P + 693,320 | $2,400 P + 392,720 |

Step 3. Set up the equation:

Part (a) $12,000\,P = 2,400\,P + 693,320$
Part (b) $12,000\,P = 2,400\,P + 392,720$

Step 4. Solve for P:

Part (a) $9,600\,P = 693,320$
$P = \$72.22$

Part (b) $9,600\,P = 392,720$
$P = \$40.91.$

Comment

The students need to be clear whether P is a monthly premium or an annual premium. An obvious source of error is to confuse these in calculating expenses. Another choice that must be made is how many policies to consider. Since the data is given in claims per thousand, 1000 policies seems the obvious choice. Two alternate approaches are sketched here for part (a).

Assume 1000 policies and P the annual premium. The balance equation for part (a) is:

$$1000\,P = (132)(5000) + (0.15)(1000)\,P + 20,000 + 12,000 + 1,320 + (0.05)(1000)\,P$$
$$800\,P = 693,320$$
$$P = \$866.65$$

Monthly premium is \$72.22.

Assume one policy. Then the claim cost becomes $(\frac{132}{1000})(5000) = \$660$. In this case $\frac{132}{1000}$ represents the probability that an individual policyholder in the category (16-19 male) will make a claim. In this case, if P is the yearly premium, the balance equation is:

$$P = (\frac{132}{1000})(5000) + 0.15\,P + 20 + 12 + (\frac{132}{1000})(10) + 0.05\,P$$
$$0.8\,P = 693.32$$
$$P = 866.65$$

Monthly premium is \$72.22.

## B. The Problem

Let $P be the monthly premium to be charged. Assume 1000 policies are issued. In this case it is necessary to take account of the time the payments are made since the present value of each payment and cost must be considered. The fundamental equation of value then makes it possible to solve for P.

The solution is carried out first for part (a) of the problem.

SOLUTION (MALE)

Step 1. Present Value of Future Premiums:

Let P = Monthly premium

$$PVFP = 1000P + \frac{1000\ P}{1.01} + \frac{1000\ P}{(1.01)^2} + \frac{1000\ P}{(1.01)^3} + \ldots + \frac{1000\ P}{(1.01)^{11}}$$

$$= 1000\ P\ (1 + v + v^2 + \ldots + v^{11})$$

where $v = \frac{1}{1.01}$

$$= 1000\ P\ \left(\frac{1 - v^{12}}{1 - v}\right)$$

$$= 1000\ P\ (11.36763)$$

$$= 11{,}367.63\ P$$

Step 2. Present Value of Future Costs:

Again, three major categories are considered, claims, expenses, and risk charge.

$$\underset{PV\ (Claims)}{\underline{Claims}} = \frac{132 \times \$5000}{(1.01)^{12}}$$

$$= \$585{,}716.49$$

Expenses

$$\text{PV (Commissions, taxes and fees)} = 0.15\left[1000P + \frac{1000P}{1.01} + \frac{1000P}{(1.01)^2} + \ldots + \frac{1000P}{(1.01)^{11}}\right]$$

$$= (0.15)(1000)(11.36763)P$$

$$\text{PV (Issuing policy)} = \$20 \times 1000 = \$20{,}000$$

$$\text{PV (Billing and management)} = 1000\left[1 + \frac{1}{1.01} + \frac{1}{(1.01)^2} + \ldots + \frac{1}{(1.01)^{11}}\right]$$

$$= \$11{,}367.63$$

$$\text{PV (Claim processing)} = \frac{132 \times \$10}{(1.01)^{12}} = \$1{,}171.43$$

$$\text{Total PV (Expenses)} = 0.15[11{,}367.63P] + \$20{,}000.00 + \$11{,}367.63 + \$1{,}171.43$$

$$= 1{,}705.14P + \$32{,}539.06$$

Risk Charge

$$\text{PV (Risk charge)} = 0.05\left[1000P + \frac{1000P}{1.01} + \frac{1000P}{(1.01)^2} + \ldots + \frac{1000P}{(1.01)^{11}}\right]$$

$$= 0.05[11{,}367.63P]$$

$$= 568.38P$$

Step 3. Set up the equation:

$$PVFP = PVFC$$

$$11{,}367.63P = \$585{,}716.49 + 1{,}705.14P + \$32{,}539.06 + 568.38P$$

Step 4. Solve for P:

$$P = \frac{\$585{,}716.49 + \$32{,}539.06}{(11{,}367.63 - 1{,}705.14 - 568.38)}$$

**P = $67.98.**

SOLUTION (FEMALE)

PVFP = PVFC

The solution is carried out along the same lines for part (b). Notice that the only changes occur in terms involving claims. This changes PV(Claims) and also one term in PV(Expenses).

Present Value of Future Premiums:

Let P = Monthly premium

$$PVFP = 1000\ P + \frac{1000\ P}{(1.01)} + \ldots + \frac{1000\ P}{(1.01)^{11}}$$

$$= 11{,}367.63\ P$$

Present Value of Future Costs:

$$\text{PV (Claims)} = \frac{72 \times \$5{,}000}{(1.01)^{12}} = \$319{,}481.72$$

Expenses

PV (Commissions, taxes, fees) = 0.15 [11,367.63 P]

PV (Issuing policy) = 20,000

PV (Billing and management) = 11,367.63

$$\text{PV (Claims Processing)} = \frac{72 \times 10}{(1.01)^{12}} = 638.96$$

Total PV (Expenses) = 1,705.14 P + 32,006.59

Risk Charge

PV(Risk Charge) = 568.38 P

PVFP = PVFC

$$11{,}367.63\ P = \$319{,}481.72 + 1{,}705.14\ P + \$32{,}006.59 + 568.38\ P$$

$$P = \frac{\$319{,}481.72 + \$32{,}006.59}{(11{,}367.63 - 1{,}705.14 - 568.38)}$$

$$\mathbf{P = \underline{\$38.65}.}$$

## C. Challenge Problem

Let P = premium

PVFP = PVFC

$$PVFP = 1000\ P\ (1 + v + v^2 + \ldots + v^{n-1})$$

$$= 1000\ P\left(\frac{1 - v^n}{1 - v}\right)$$

PVFC = PV(Claims) + PV(Expenses) + PV(Risk charge)

$$PV(\text{Claims}) = (NA)(CA)v^n$$

PV(Expenses) = PV(Commissions, taxes and fees)
+ PV(Issuing policy)
+ PV(Billing and management)
+ PC(Claim processing)

$$PV(\text{Commissions, taxes and fees}) = (1000)(E)(P)\left(\frac{1 - v^n}{1 - v}\right)$$

$$PV(\text{Issuing policy}) = 1000(EI)$$

$$PV(\text{Billing and management}) = 1000(EP)\left(\frac{1 - v^n}{1 - v}\right)$$

$$PV(\text{Claim processing}) = (NA)(EC)v^n$$

$$PV(\text{Expenses}) = (1000)(E)(P)\left(\frac{1 - v^n}{1 - v}\right) + 1000(EI) + 1000(EP)\left(\frac{1 - v^n}{1 - v}\right) + (NA)(EC)v^n$$

$$PV(\text{Risk charge}) = (1000)(RR)(P)\left(\frac{1 - v^n}{1 - v}\right)$$

Set PVFP = PVFC and solve for P.

$$1000P\left(\frac{1 - v^n}{1 - v}\right) = (NA)(CA)v^n + (1000)(E)(P)\left(\frac{1 - v^n}{1 - v}\right) + 1000(EI) + (1000)(EP)\left(\frac{1 - v^n}{1 - v}\right) + (NA)(EC)v^n + (1000)(RR)(P)\left(\frac{1 - v^n}{1 - v}\right)$$

$$1000P\left(\frac{1 - v^n}{1 - v}\right)(1 - E - RR) = (NA)v^n[(CA)+(EC)] + 1000\left[(EI)+(EP)\left(\frac{1 - v^n}{1 - v}\right)\right]$$

$$P = \frac{(NA)[(CA)+(EC)](v^n) + 1000\left[(EI)+(EP)\left(\frac{1 - v^n}{1 - v}\right)\right]}{1000\ (1 - E - RR)\left(\frac{1 - v^n}{1 - v}\right)}$$

Test the formula for 16-19 year old males:

NA = 132 EI = \$20
CA = \$5000 EP = \$1
I = 0.12 EC = \$10
N = 12 RR = 0.05
E = 0.15 $v = \frac{1}{1 + \frac{I}{N}} = \frac{1}{1.01}$

$$P = \frac{(132)(\$5010)(v)^{12} + 1000[\$20 + \$1(\frac{1 - v^{12}}{1 - v})]}{1000(1 - 0.15 - 0.05)(\frac{1 - v^{12}}{1 - v})}$$

$$= \frac{586{,}887.92 + 31{,}367.63}{9094.10}$$

$$= \$67.98$$

A similar check can be made for 16-19 year old females.

# VII. The Computer Program

A computer diskette is available with the AIM packet. Included on the diskette is a solution program written for the Apple II computer. After booting up the AIM diskette, the program can be run by typing RUN PRICING AUTO INSURANCE. The student is now ready to enter data necessary to compute premiums per pay period for a specific case. The necessary data to be entered includes: number of accidents per 1000 drivers; average claim cost per accident; interest rate per year; and the number of pay periods per year. The computer calculates and prints the resulting premium. The student is then given the option of calculating the premium with different statistics without exiting the program. This serves comparative as well as informative purposes.

Although a solution program is included with the AIM packet, students should be encouraged to write their own programs. To facilitate this, a flow chart and program listing have been included in the appendix. The students can use their imagination in writing their own program or in enhancing the given solution program.

# VIII. The Written Report

The ability to write is an important skill in any area. Report writing is a regular part of the job of a person working in industry. In the AIM problem the student is playing the role of an employee of American Fidelity Assurance. This role includes writing a report stating the work requested, the work done, and the results found. In this case the report should deal with Section V-B, The Problem. Encourage the student to be brief, precise, and clear. The employer wants to know what the answer is, and how the student got that answer, not how hard it was and what false starts were made. Short but complete sentences are preferred. Spelling and punctuation should be correct. A suggested format is given in the appendix.

Although students are rarely delighted at the prospect of writing a report, they are very proud when they see what they have accomplished! Suggest that they keep it for their portfolio to use when applying for scholarships or jobs.

# IX. Additional Questions and Projects

## A. Think About Percent

1. A store is offering 20% off its dresses. What would be the price of a $48 dress? What would your total bill be if the sales tax in your state is 6%?

2. A restaurant suggests that the waitress receive a tip of 15% of the bill. You and your friends paid $19.31 for your food. How much should you leave as a tip?

3. Out of 741 field goal attempts, a basketball team made 384 field goals. Out of 320 free throw attempts, 211 were successful. What is their percent from the field, and what is their percent from the line?

4. One "clever" basketball coach stated that his team had a perfect game because they shot 100%, --40% from the field, and 60% from the line. Do you agree with his mathematics?

5. The information in the table, Section IV of the Student Resource Book, says that male drivers under 20 had 132 accidents per thousand, and male drivers from 20 to 24 had 95 accidents per thousand. What percent of male drivers under 20 had accidents? What percent of the drivers 20 to 24 had accidents? Have you enough information to say what percent of male drivers 24 and under had accidents?

6. A storewide after-Christmas sale marks down all items 20%. By the end of January, some items still unsold are marked down an additional 30%. Does this mean the total mark down is 50%? If not, is it more or less than 50%? Check your guess by calculating the total percent of mark down.
(Hint: find the reduced price of a $100 article).

## B. Think About the Geometric Series

1. Which of the following series are geometric series?

   (a) 3 + 9 + 27 + 81 + 243

   (b) 3 - 9 + 27 - 81 + 243

   (c) 3 + 6 + 9 + 12 + 15

   (d) 2 + 6 + 18 + 54 + 162

   (e) 2 + 2 + 2 + 2 + 2

2. The following are geometric series:

I. $4 + 8 + 16 + 32 + 64 + 128$

II. $5 + 0.5 + 0.05 + 0.005 + 0.0005$

III. $3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$

(a) Write a, r, and n, in each case.

(b) For which series can we use the sum formula

$$S_n = \frac{a(1 - r^n)}{1 - r}$$ to find the sum?

(c) Find the sum in each case.

3. Each of the following series has r = 1. Find the sum in each case.

(a) $S_2 = 5 + 5$

(b) $S_3 = 5 + 5 + 5$

(c) $S_7 = 5 + 5 + 5 + 5 + 5 + 5 + 5$

(d) $S_n = 5 + 5 \ldots + 5$ (n terms)

4. Write a formula for $S_n$, where $S_n = a + a + \ldots + a$ (n terms).

5. Suppose $S_5 = 3 - 3 + 3 - 3 + 3$.

(a) Is this a geometric series? What is a? What is r? What is n?

(b) Write $S_6$.

(c) For what n is the last term in the series -3?

6. $S_n = a - a + a - \ldots + (-1)^{n-1} a$.

(a) Find the value of the sums $S_2$, $S_4$, $S_6$, $S_8$.

(b) Find the value of the sums $S_3$, $S_5$, $S_7$, $S_9$.

(c) Write an expression for $S_n$.

7. $S_n = 3 + \frac{3}{2} + \frac{3}{2^2} + \ldots + \frac{3}{2^{n-1}}$

(a) Find the value of the sum $S_3$.

(b) Find the value of the sum $S_{10}$.

(c) Find the value of the sum $S_{20}$.

(d) Guess a value for $S_{100}$.

(e) On your hand calculator, how large is n before you cannot tell the difference between $S_n$ and $S_{n+1}$?

8. $S_n = 3 + 3(2) + 3(2)^2 + 3(2)^3 + \ldots + 3(2)^{n-1}$.

(a) Find the value of $S_3$.

(b) Find the value of $S_{10}$.

(c) What happens to the values of $S_n$ as n gets larger and larger?

## C. What if...

The economy can change rapidly in many ways. Any of these can affect the cost of insurance. Your computer program can help you investigate the effect of some of these changes.

1. Suppose the interest rate changes from 1% per month to 1.25% per month, with no other changes in the economy. What will the new monthly premium be for males under 20? What will it be for females under 20?

2. Suppose the interest rate falls from 12% per year to 9% per year, with no other changes. What will the new monthly premium be for males under 20? What will it be for females under 20?

3. Rising interest rates are sometimes accompanied by rising costs. Suppose the interest rate becomes 1.25% per month, and at the same time the average claim per accident is $6000. What will the new monthly premium be for males under 20? What will it be for females under 20?

4. In the problem, Section V-B, it was assumed that premiums were paid monthly. Suppose that premiums were paid quarterly, with the interest rate still at 1% per month. What would be the quarterly payment? Is this more or less than paying monthly?

Suppose the premiums are paid semiannually. What is the payment? Which do you think would be the best way to pay and why?

5. According to the data provided, the number of accidents per thousand for male drivers in the 60-64 age group is 0.288 times the number of accidents per thousand for male drivers in the 16-19 age group. Will their premium be 0.288 times the premium for the younger drivers? Check your guess by calculation.

6. Calculate the monthly premium for each age group of male drivers. Draw a bar graph to illustrate the results. In what age group is the premium lowest?

## D. Think About Interest

1. The annual interest rate for two certificates of deposit (CD's) is 10%. CD #1 compounds interest annually, while CD #2 compounds interest quarterly.

   (a) If $10,000 is the initial amount of both CD's, find the difference in their values after 2 years.

   (b) If $N is the initial amount of both CD's, find the difference in their values after 2 years.

2. If you have a computer, write a program to calculate the future value of $A after N compounding periods per year, with interest rate I per year. Using the program, find the following:

   (a) The future value of $1000 compounded quarterly for two years at 12% per year.

   (b) The future value of $1000 compounded monthly for two years at 12% per year.

   (c) The future value of $1000 compounded daily for two years at 12% per year.

3. Using the computer program created in 2, compare the future value of $1000 at 9% per year for 5 years, compounding quarterly, monthly, or daily.

4. How long will it take $1000 to double in value if it is invested at 9% per year, compounded quarterly? (You can use a guess and guess again technique to settle this if you wish.)

## E. Think About Present Value

Suppose you wish to borrow $5000 to buy a car. You agree to repay the loan in monthly payments over three years. Naturally you will need to pay interest on this loan, perhaps as high as 15%. You will be making payments every month. Part of the payment will cover the interest for that month, and part will begin paying back the principal. In the first month the interest is easy to calculate. The interest rate per month is 0.15/12 = 0.0125. For the first month the interest is

$$(0.0125)(5000) = \$62.50.$$

The rest of your payment will be principal. Next month your interest will be a bit less, because the principal will be less. How much less? Well, we cannot answer that until we know what the payment is. It looks as if we are going around in circles. Let's do some mathematical modeling -- which is just

another word for careful thinking. Cover the right hand column ad see if you can supply the answers to the "thinking questions" yourself.

Call the monthly payment $X.

| | |
|---|---|
| How many payments? | (12)(3) = 36 |
| How much money now? | $5000 |
| What do we call the value now of future payments? | Present Value |
| What is the present value of $B at interest rate I for N compounding periods? | $PV(B) = \frac{B}{(2+I)^N}$ |

(Look back at Section II in the Student book if you need to.)

| | |
|---|---|
| What is the value now of our future payments? | $5000 |
| What is our mathematical model for this question? | PV(all payments) = $5000 |
| What should we calculate? | PV(all payments) |
| What is I in our case? | 0.0125 |
| What is N in our case? | The biggest N is 36, but N is different for each payment. |
| When are payments made? | At the **end** of each month. |
| PV(first payment)? | I = 0.0125, N = 1<br>$PV(\text{payment } 1) = \frac{X}{1.0125}$ |
| PV(second payment)? | I = 0.0125, N = 2<br>$PV(\text{payment } 2) = \frac{X}{(1.0125)^2}$ |
| PV(third payment)? | $PV(\text{payment } 3) = \frac{X}{(1.0125)^3}$ |
| We don't want to do this 36 times! | PV(all payments) = $\frac{X}{1.0125} + \frac{X}{(1.0125)^2} + \ldots + \frac{X}{(1.0125)^{36}}$ |
| This is a geometric series. What is a? | a = X/1.0125 |
| What is r? | r = 1/1.0125 |

What is n?

What is the sum?

$n = 36$

$$\text{Sum} = \frac{X}{1.0125}\left(\frac{1 - \left(\frac{1}{1.0125}\right)^{36}}{1 - \frac{1}{1.0125}}\right)$$

$$= 28.8473X$$

Now what does our model tell us?

$28.8473X = 5000$

What is the monthly payment?

$X = 5000/28.8473$
$= \$173.33$

Let's see how the first payments work:

| Principal P | Interest .0125P | Int. to date | Paid on Principal Pd = X-Int | New Principal P - Pd |
|---|---|---|---|---|
| 5000 | 62.50 | 62.50 | 110.83 | 4889.17 |
| 4889.17 | 61.12 | 123.62 | 112.22 | 4776.95 |
| 4776.95 | 59.71 | 183.33 | 113.62 | 4663.33 |
| 4663.33 | 58.29 | 241.62 | 115.04 | 4548.29 |

We need a computer to do this! Here is the print out. I have not rounded to the nearest cent.

| Period | Interest 0.0125 P | Interest to date | Paid on Principal | New Principal |
|---|---|---|---|---|
| 1 | 62.5 | 62.5 | 110.827 | 4889.17 |
| 2 | 61.1147 | 123.615 | 112.212 | 4776.96 |
| 3 | 59.712 | 183.327 | 113.615 | 4663.35 |
| 4 | 58.2918 | 241.619 | 115.035 | 4548.31 |
| 5 | 56.8539 | 298.472 | 116.473 | 4431.84 |
| 6 | 55.398 | 353.87 | 117.929 | 4313.91 |
| 7 | 53.9239 | 407.794 | 119.403 | 4194.51 |
| 8 | 52.4313 | 460.226 | 120.896 | 4073.61 |
| 9 | 50.9201 | 511.146 | 122.407 | 3951.2 |
| 10 | 49.39 | 560.536 | 123.937 | 3827.27 |
| 11 | 47.8408 | 608.377 | 125.486 | 3701.78 |
| 12 | 46.2722 | 654.649 | 127.055 | 3574.72 |
| 13 | 44.6841 | 699.333 | 128.643 | 3446.08 |
| 14 | 43.076 | 742.409 | 130.251 | 3315.83 |
| 15 | 41.4479 | 783.857 | 131.879 | 3183.95 |
| 16 | 39.7994 | 823.656 | 133.528 | 3050.42 |
| 17 | 38.1303 | 861.786 | 135.197 | 2915.23 |
| 18 | 36.4403 | 898.227 | 136.887 | 2778.34 |
| 19 | 34.7293 | 932.956 | 138.598 | 2639.74 |
| 20 | 32.9968 | 965.953 | 140.33 | 2499.41 |
| 21 | 31.2427 | 997.195 | 142.084 | 2357.33 |
| 22 | 29.4666 | 1026.66 | 143.86 | 2213.47 |
| 23 | 27.6683 | 1054.33 | 145.659 | 2067.81 |
| 24 | 25.8476 | 1080.18 | 147.479 | 1920.33 |
| 25 | 24.0041 | 1104.18 | 149.323 | 1771.01 |
| 26 | 22.1376 | 1126.32 | 151.189 | 1619.82 |
| 27 | 20.2477 | 1146.57 | 153.079 | 1466.74 |
| 28 | 18.3342 | 1164.9 | 154.993 | 1311.75 |
| 29 | 16.3968 | 1181.3 | 156.93 | 1154.82 |
| 30 | 14.4352 | 1195.73 | 158.892 | 995.923 |
| 31 | 12.449 | 1208.18 | 160.878 | 835.046 |
| 32 | 10.4381 | 1218.62 | 162.889 | 672.157 |
| 33 | 8.40196 | 1227.02 | 164.925 | 507.232 |
| 34 | 6.3404 | 1233.36 | 166.987 | 340.245 |
| 35 | 4.25306 | 1237.62 | 169.074 | 171.171 |
| 36 | 2.13964 | 1239.76 | 171.187 | -.0163574 |

Notice that this print out shows you have paid $1239.75 in interest, as well as the $5000 originally borrowed, a total of $6239.75. Probably it is worth this to you, but the cost of interest payments is something you should think about carefully.

1. Do the following problem using a hand calculator only. Suppose you borrow $1000 for four months at an interest rate of 18% per year. You repay in four monthly payments at the end of each month.

   (a) What is your monthly payment?

   (b) Make a table showing:
   Principal owed at the first of the month
   Total interest up to that time
   Amount of the payment that reduces principal
   New principal owed after the payment.

   (c) What should your last entry be?

2. Do problem 1 part (a) assuming that you repay the loan in eight months.

3. Look at the print out for the example.

   (a) How many months did it take for the interest to be less than 1/3 of the monthly payment?

   (b) How much was paid out in interest altogether?

   (c) Make a graph illustrating the principal outstanding during the loan repayment. Plot the data every third month.

4. A loan of $A runs for N periods per year, at interest rate I per year for Y years. Write a computer program which will calculate the amount of loan payment per period, $X. Have your program make a table showing the amount of each payment which is interest, the interest paid up to that time, the amount of the payment which reduces the principal, and the outstanding principal at each payment.

5. Use your program to get the following information: Payment per period, the amount of each payment which is interest, the total interest paid up to that time, the amount of the payment which is a payment on principal, and the outstanding principal after each payment.

   (a) A = $5000, I = 0.16, and payments are made quarterly for five years.

   (b) A = $5000, I = 0.16, and payments are made twice a year for five years.

   (c) A = $5000, I = 0.16, and payments are made annually for five years.

By which method of payment is the most interest paid? What is the reason for this?

6. Study the effect of interest rate changes on the monthly payments by working the three cases in problem 5.

(a) Using an interest rate of 12% per year instead of 16% per year.

(b) Using an interest rate of 9% per year.

7. Suppose you decide to pay off the $5000 loan in 4 years rather than 5. Would this increase or decrease your payments per period? Would this increase or decrease the total amount of interest paid? Assume the interest rate is 12% per year. How much is saved by paying the loan in 4 years? What about paying the loan in 3 years? What is the total amount paid out for the loan in this case?

Discuss which length of time you would choose to pay off this $5000 loan. Give reasons for your decision.

## F. Answers

### part A

1. Sale price of dress is $38.40; total bill is $40.70.

2. Suggested tip is $2.90. How about leaving $3.00?

3. From the field 51.82%; from the line 65.94%.

4. No. You cannot combine the percents by addition. Clearly, the team was not perfect in either category.

5. Male drivers under 20: 13.2%
   Male drivers 20-24: 9.5%

   You would need to know how many drivers there were in each category. For example, suppose there are 10,000 male drivers under 20 and 20,000 male drivers 20-24. Then there are 1320 + 2900 = 3220 accidents for 30,000 drivers, that is, 107 per thousand, or 10.7%. On the other hand, if there were 30,000 male drivers under 20 and 20,000 male drivers in the 20-24 category, there would be 3960 + 1900 = 5860 accidents for 50,000 drivers, or 117 accidents per thousand, that is 11.7%.

6. The mark down is less than 50%. It is 44%. A $100 article is first marked down to $80, then the $80 article is reduced by $24 to $56, making the combined reduction of $44.

### part B

1. All are geometric except (c). In (a), $a = 3$, $r = 3$; in (b) $a = 3$, $r = -3$; in (d) $a = 2$, $r = 3$, in (e) $a = 2$, $r = 1$.

2. (a). I: $a = 4$, $r = 2$, $n = 6$; II: $a = 5$, $r = 0.1$, $n = 5$; III: $a = 3$, $r = 1$, $n = 10$.

   (b) I and II

   (c) I $\dfrac{4(1 - 2)^6}{1 - 2} = \dfrac{4(-63)}{-1} = 252$

   II $\dfrac{5(1 - (0.1)^5)}{1 - 0.1} = \dfrac{5(1 - 0.00001)}{0.9} = 5.5555$

   III 30

3. (a) $(5)(2) = 10$; (b) $(5)(3) = 15$;
   (c) $(5)(7) = 35$; (d) $(5)(n) = 5n$

4. $S_n = an$

(a) Yes, a = 3, r = -1, n = 5

(b) $S_6 = 3 - 3 + 3 - 3 + 3 - 3$

(c) The last term is -3 if n is even; 3 if n is odd.

6. (a) 0 in each case

(b) a in each case

(c) 0 if n is even, a if n is odd; or

$$S_n = \frac{a(1 - (-1)^n)}{2}$$

Note that $1 - (-1)^n = 0$ if n is even and 2 if n is odd.

7. (a) $S_3 = 5.25$, (b) $S_{10} == 5.994190$, (c) $S_{20} = 5.999994$, (d) $S_{100} = 6$, (e) The answer depends on the calculator. $S_{30} = 6$ accurate to 8 decimal places, $S_{32} = 6$ accurate to 9 decimal places.

8. (a) $S_3 = 21$

(b) $S_{10} = 3069$

(c) $S_n$ increases without bound.

## part C

1. males $66.96
   females $38.11

2. males $69.02
   females $39.20

3. males $79.64
   females $45.02

4. quarterly payments: males $199.62
   females $112.42

   Total monthly premiums are more than total quarterly payments.

   Semiannual payments: males $392.69
   females $220.61

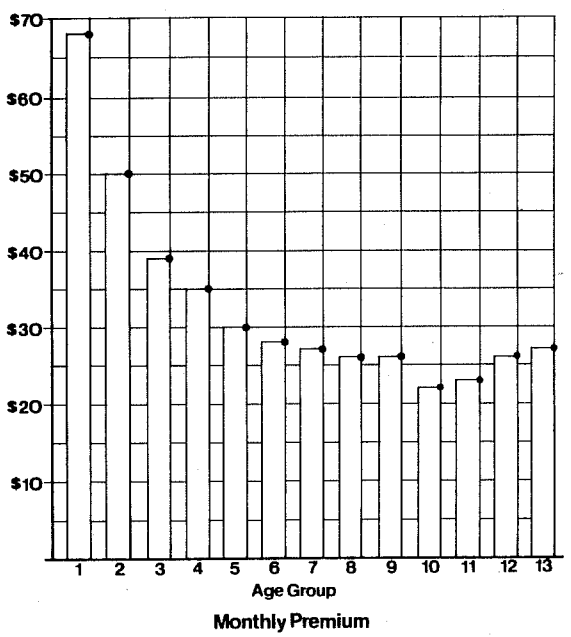
5.

| | monthly premium |
|---|---|
| males age 60-64 | $22.03 |
| males age 16-19 | $67.98 |

Premiums for male drivers in the 60-64 age group are 0.324 times the premiums for 16-19 year old males.

6. The premiums for the male age groups are:

| Age Group | Premium |
|---|---|
| 1. under 20 | $67.98 |
| 2. 20-24 | 49.89 |
| 3. 25-29 | 39.14 |
| 4. 30-34 | 35.23 |
| 5. 35-39 | 29.36 |
| 6. 40-44 | 28.38 |
| 7. 45-49 | 27.89 |
| 8. 50-54 | 26.43 |
| 9. 55-59 | 26.43 |
| 10. 60-64 | 22.03 |
| 11. 65-69 | 23.98 |
| 12. 70-74 | 26.43 |
| 13. 75 & over | 26.92 |



## part D

1. (a) CD #1 has value $12,100; CD #2 has value $12,184.03; the value of CD #2 is $84.03 more than the value of CD #1.
   (b) 0.008403 N

2. (a) $1266.77
   (b) $1269.73
   (c) $1271.20

   This program is included on the AIM diskette, and titled, "Future Value".

3. Compounded quarterly $1560.51
   monthly $1565.68
   daily $1568.23

4. Guess Y = 10 $2435.19 (too big)
   Y = 8 $2038.10 (still too big)
   Y = 7 $1864.54 (too small)
   Y = 7.75 $1993.25 (not quite big enough)

Since compounding interest quarterly is a discrete process, it would take 8 years for the value to double.

## part E

1. (a) $259.45

   (b)

| Month | Principal | Interest | Sum of Interest | Paid on Principal | New Principal |
|---|---|---|---|---|---|
| 1 | 1000.00 | 15.00 | 15.00 | 244.45 | 755.55 |
| 2 | 755.55 | 11.33 | 26.33 | 248.11 | 507.44 |
| 3 | 507.44 | 7.61 | 33.94 | 251.83 | 255.61 |
| 4 | 255.61 | 3.83 | 37.77 | 255.61 | 0 |

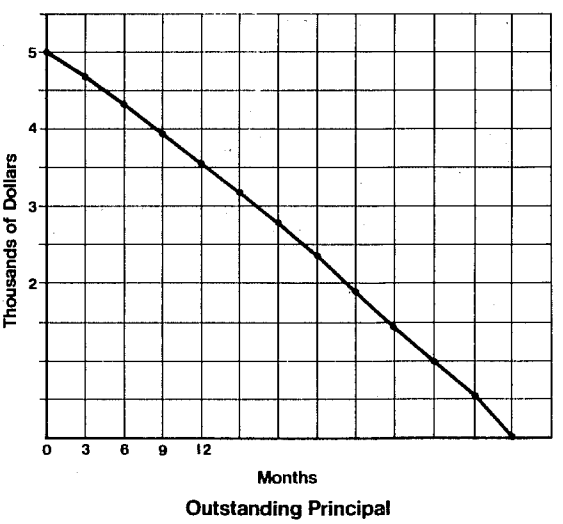
   (Answers may differ by a few cents because of rounding.)

   (c) 0

2. Payment per period $133.58

3. (a) Monthly payment $173.33
   Interest is less than $57.78 after 4 months.

   (b) $1239.76 was paid out in interest.



This program is on the AIM diskette titled, "Loan Payment". Note that the program rounds to cents at each step.

5. (a) Quarterly payment $367.91
Total interest paid $2358.14

(b) Semiannual payment $745.18
Total interest paid $2451.46

(c) Annual payment $1527.05
Total interest paid $2635.22

6. Interest rate 12%

| | Quarterly | Semi-annually | Annually |
|---|---|---|---|
| Payment per period | $336.08 | $679.34 | $1387.05 |
| Total interest | $1721.57 | $1793.40 | $1935.24 |

Interest rate 9%

| | Quarterly | Semi-annually | Annually |
|---|---|---|---|
| Payment per period | $313.21 | $631.90 | $1285.46 |
| Total interest | $1264.21 | $1318.94 | $1427.31 |

7. Shortening the number of years increases the payment per period and decreases the total amount of interest paid.

| | Y = 5 | Y = 4 | Y = 3 |
|---|---|---|---|
| Quarterly Payment | $336.08 | $398.06 | $502.31 |
| Total interest | $1721.57 | $1368.87 | $1027.72 |
| Total paid | $6721.57 | $6368.87 | $6027.72 |

The total amount saved is $693.85.

The discussion might include in favor of 5 years:
1) smaller monthly payments
2) interest used as a deduction in income tax

In favor of 3 years:
1) total cost of the car is considerably less
2) you would be free of payments earlier
3) the title would be clear if you desired to trade the car.

# X. References

The books listed here are a few of many books which relate to business mathematics in general and different types of insurance.

For mathematical background any high school algebra would be satisfactory, for example:

Benice, Daniel D., Arithmetic and Algebra, 2nd Edition, Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1979.

If you wish more about geometric series, use any high school calculus text, for example:

Thomas, George B., Jr., Elements of Calculus and Analytic Geometry, 2nd Edition, Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1972.

For a general background in using mathematics in a business setting, any high school text in business math would be satisfactory, for example:

Bello, Ignacio, Contemporary Business Mathematics, Philadelphia: W. B. Saunders Company, 1975.

Kravitz, Wallis and Vincent Brant, Consumer Related Mathematics, a Business Approach, New York: Holt, Rinehart and Winston, Inc., 1971.

Lewis, Harry, Mathematics for Daily Living, Cincinnati, Ohio: McCormick-Mathers Publishing Company, Inc., 1970.

For a good discussion of the general principles of auto insurance:

Vaughan, Emmett J., Fundamentals of Risk and Insurance, Fourth Edition, Chapters 30, 31, 32. New York: John Wiley and Sons, Inc., 1986.

It is unlikely that you will find texts which follow exactly the development in this module. It is good for students to realize that there are many points of view from which the same problems can be approached. Many books base their work on tables. In this presentation, we have chosen to avoid tables and emphasize the basic mathematics behind them.

# XI. Appendix

## A. Glossary

**balance principle** - total income = total costs.

**claim** - a demand for something due or believed to be due.

**claim cost** - the amount paid out in settling claims.

**commission** - the amount paid to the seller for insurance sold.

**compound interest** - simple interest repeatedly applied to a sum increased by the simple interest earned in each time period.

**fundamental equation of value** - present value of future premiums = present value of future costs.

**future value** - the principal amount plus the interest it has earned; the future value of $P, after n compounding periods, at interest rate I per period is

$$P(1 + I)^n.$$

**geometric sequence** - a set of terms of the form a, ar, $ar^2$, ... where each term is formed from the preceding term by multipying by r.

**geometric series** - the sum of the first n terms of a geometric sequence;

$$S_n = a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, \quad r \neq 1.$$

**interest** - the amount paid or received by a person or institution to use or borrow the money of others.

**interest rate** - a number used to determine the amount of interest, usually stated as a percent of the principal for a given time period.

**policy** - a legal document which specifies in detail the contract of insurance for a particular time period.

**present value of a payment** - the value today of a payment to be made in the future.

**principal** - the amount on which interest is calculated.

**risk charge** - an amount added to costs by an insurance company as a margin of safety to cover unforeseen expenses and costs.

**simple interest** - the product of principal, interest rate, and time.

## B. Report Format

**OPENING MATERIAL**

| | |
|---|---|
| Title Page | |
| Table of Contents | Section headings and page numbers. |
| Glossary | This should include only terms special to this problem. |
| List of Symbols | |
| Abstract | A brief statement of what the student was asked to do and the answer. (No more than four or five lines.) |

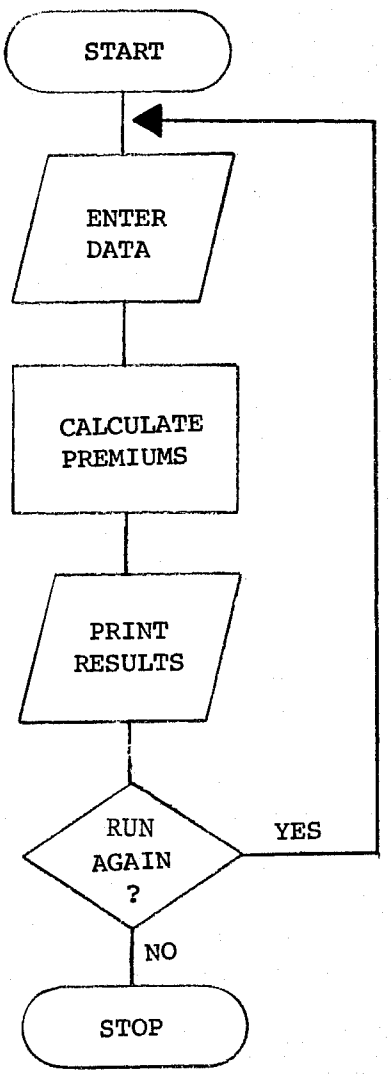
**BODY**

| | |
|---|---|
| Introduction | Statement of what was assigned and why it was needed. |
| Discussion | The simplifying assumptions. An outline of the formulas given and any formulas that were derived, and an outline of the procedure. |
| Results | A more detailed statement of the results than that given in the abstract. |
| Conclusions and Recommendations | If any value judgment can be given, it should be included here. |

**APPENDIX**

| | |
|---|---|
| References and Acknowledgements | Any books or articles consulted. Any people who helped. |
| Computation | A flow chart or program if needed. Detailed results or calculations. |

## C. Flowchart and Program



```
]LIST

 100  REM       PRICING AUTO INSURANCE
 110  REM
 120  REM       WRITTEN BY JEFF DIMICK
 130  REM            DECEMBER 1985
 140  REM
 150  REM       VARIABLE DECLARATION:
 160  REM
 170  REM    NA=NUMBER OF ACCIDENTS/1000
 180  REM    CA=AVERAGE CLAIM COST/ACCIDENT
 190  REM     I=INTEREST RATE/YEAR
 200  REM    N=NUMBER OF PAY PERIODS/YEAR
 210  REM    E=EXPENSE/POLICY (%)
 220  REM    EI=COST OF ISSUING POLICY
 230  REM    EP=COST OF PREMIUM BILLING & MGT/POLICY
 240  REM    EC=COST OF CLAIM PROCESSING/CLAIM
 250  REM    RR=RISK OBJECTIVE (%)
 260  REM    P=PREMIUM/PAY PERIOD
 270  REM
 280  REM    ALL PERCENTS ARE REPRESENTED IN
 290  REM    DECIMAL NOTATION.
 300  REM
 310  HOME : VTAB (8): PRINT  TAB( 7);"APPLICATIONS IN MATHEMATICS": PRINT
      : PRINT : PRINT  TAB( 10);"PRICING AUTO INSURANCE": VTAB (20): PRINT
       TAB( 8);"TYPE <RETURN> TO CONTINUE";: GET A$
320 C$ = "                                                "
 330  REM
340 E = .15:EI = 20:EP = 1:EC = 10:RR = .05
 350  REM
 360  REM         ENTER DATA
 370  REM
 380  HOME : PRINT  TAB( 10);"PRICING AUTO INSURANCE": PRINT : PRINT : PRINT
       TAB( 17);"INPUTS": PRINT : PRINT
 390  PRINT "  (1)  NUMBER OF ACCIDENTS/1000 ";: HTAB (35): INPUT AN$
400 NA =  VAL (AN$)
 410  IF NA < 0 OR NA > 1000 THEN  FLASH : PRINT " THE NUMBER MUST BE BETW
     EEN 0 AND 1000": NORMAL : VTAB (7): GOTO 390
 420  PRINT C$: PRINT "  (2)  AVG. CLAIM COST/ACCIDENT";: HTAB (35): INPUT
     AN$
430 CA =  VAL (AN$)
 440  PRINT : PRINT "  (3)  INTEREST RATE/YR. (DECIMAL)";: HTAB (35): INPUT
     AN$
450 I =  VAL (AN$)
 460  IF I <  = 0 THEN  FLASH : PRINT " THE INTEREST RATE MUST BE POSITIVE
     ": NORMAL : VTAB (10): GOTO 440
 470  PRINT C$
 480  PRINT "  (4)  NUMBER OF PAY PERIODS";: HTAB (35): INPUT AN$
490 N =  VAL (AN$)
 500  IF N <  = 0 THEN  FLASH : PRINT " THE NUMBER OF PERIODS MUST BE POSI
     TIVE": NORMAL : VTAB (13): GOTO 480
 510  PRINT C$ + " "
 520  REM
 530  REM   CHECK VALUES
 540  VTAB (20): PRINT  TAB( 6);"ARE THESE VALUES CORRECT?";: GET A$
 550  IF A$ = "Y" THEN 650
 560  IF A$ <  > "N" THEN  PRINT : PRINT  TAB( 10);"PLEASE ENTER Y OR N": GOTO
     540
 570  PRINT : PRINT  TAB( 6);"ENTER NUMBER TO BE CHANGED ";: GET A$: VTAB
```

```
     (20): PRINT C$ + "   "
580  IF A$ <  > "1" AND A$ <  > "2" AND A$ <  > "3" AND A$ <  > "4" THEN
      VTAB (20): GOTO 570
590  IF A$ = "1" THEN  VTAB (7): HTAB (35): INVERSE : PRINT "      ": NORMAL
     : VTAB (7): HTAB (35): INPUT AN$:NA =  VAL (AN$)
600  IF NA < 0 OR NA > 1000 THEN  FLASH : PRINT " THE NUMBER MUST BE BETW
     EEN 0 AND 1000": NORMAL : VTAB (7): GOTO 590
610  IF A$ = "2" THEN  VTAB (9): HTAB (35): INVERSE : PRINT "      ": NORMAL
     : VTAB (9): HTAB (35): INPUT AN$:CA =  VAL (AN$)
620  IF A$ = "3" THEN  VTAB (11): HTAB (35): INVERSE : PRINT "      ": NORMAL
     : VTAB (11): HTAB (35): INPUT AN$:I =  VAL (AN$)
630  IF A$ = "4" THEN  VTAB (13): HTAB (35): INVERSE : PRINT "      ": NORMAL
     : VTAB (13): HTAB (35): INPUT AN$:N =  VAL (AN$)
640  GOTO 530
650  HOME : VTAB (10): PRINT  TAB( 6);"CALCULATING, PLEASE WAIT!!"
660  REM
670  REM   MAKING CALCULATIONS
680  REM
690 V = 1 / (1 + I / N)
700 P = (NA * CA * V ^ N + EI * 1000 + NA * EC * V ^ N + EP * 1000 * (1 -
     V ^ N) / (1 - V)) / (1000 * (1 - V ^ N) / (1 - V) * (1 - RR - E))
710  REM    ROUND TO TWO DECIMAL PLACES
720 P =  INT (P * 100 + .5) / 100
730  HOME : PRINT  TAB( 10);"PRICING AUTO INSURANCE": VTAB (10): PRINT  TAB(
     8);"THE CALCULATED PREMIUM FOR": PRINT  TAB( 8);"EACH PAY PERIOD IS
     $";P
740  VTAB (20): PRINT  TAB( 6);"WOULD YOU LIKE TO MAKE ANOTHER ": PRINT  TAB(
     6);"RUN WITH DIFFERENT INPUTS?";: GET A$
750  IF A$ <  > "N" AND A$ <  > "Y" THEN  PRINT : PRINT  TAB( 10);"PLEASE
      ENTER Y OR N": GOTO 740
760  IF A$ = "Y" THEN 360
770  HOME
780  END
```

```
]
]LIST

100  HOME : PRINT  TAB( 08);"FUTURE VALUE OF A"
110  PRINT : PRINT : INPUT "ENTER THE AMOUNT A ";A
120  PRINT : PRINT "ENTER THE NUMBER OF COMPOUNDING PERIODS": INPUT "PER
     YEAR ";N
130  PRINT : INPUT "ENTER THE INTEREST RATE/YR (DECIMAL) ";I
140  PRINT : INPUT "ENTER THE NUMBER OF YEARS ";Y
150  REM : I/N IS THE INTEREST RATE PER COMPOUNDING PERIOD
160  REM : Y*N IS THE NUMBER OF COMPOUNDING PERIODS
170  PRINT : PRINT : PRINT "ARE THESE VALUES CORRECT? ";: GET A$
180  IF A$ = "N" THEN 100
190  IF A$ <  > "Y" THEN 170
200  REM : CALCULATE FUTURE VALUE (FV):
210 FV = A * (1 + I / N) ^ (Y * N)
220  PRINT : PRINT : PRINT : PRINT "THE FUTURE VALUE OF A IS ";FV
230  PRINT : PRINT : PRINT "WOULD YOU LIKE TO RUN AGAIN? ";: GET A$
240  IF A$ = "Y" THEN 100
250  IF A$ <  > "N" THEN 230
260  END
```

```
]
]LIST

100  REM :         LOAN PAYMENTS
110  REM :
120  REM :     WRITTEN FEBRUARY, 1986
130  REM :
140  REM :  VARIABLE DECLARATION:
150  REM : P = PRINCIPAL
160  REM : N = NUMBER OF PAY PERIODS/YEAR
170  REM : I = INTEREST RATE/YEAR (DECIMAL)
180  REM : Y = NUMBER OF YEARS
190  REM : R = INTEREST
200  REM : PD = PAID PRINCIPAL
210  REM : NP = NEW PRINCIPAL
220  REM : SUM = SUM OF INTEREST
230  REM : X = PAYMENT PER PAY PERIOD
240  HOME : PRINT  TAB( 8);"LOAN PAYMENTS"
250  PRINT : PRINT : INPUT "ENTER PRINCIPAL ";P
260  PRINT : INPUT "ENTER NUMBER OF PAY PERIODS/YEAR ";N
270  PRINT : INPUT "ENTER INTEREST RATE/YEAR (DECIMAL) ";I
280  PRINT : INPUT "ENTER NUMBER OF YEARS FOR THE LOAN ";Y
290  REM :
300  PRINT : PRINT : PRINT "ARE THESE VALUES CORRECT? ";: GET A$
310  IF A$ = "N" THEN 100
320  IF A$ <  > "Y" THEN 300
330  REM : CALCULATE PAYMENT/PAY PERIOD:
340 X = P * (I / N) / (1 - 1 / (1 + I / N) ^ (N * Y))
350 X =  INT (X * 100 + .5) / 100
360 SUM = 0
370  HOME : PRINT  TAB( 15);"RESULTS": PRINT : PRINT : PRINT
380  FOR K = 1 TO N * Y
390 R = I / N * P
400 PD = X - R
410 NP = P - PD
420 SUM = SUM + R
430  REM :  ROUND RESULTS:
440 R =  INT (R * 100 + .5) / 100:PD =  INT (PD * 100 + .5) / 100:NP =  INT
    (NP * 100 + .5) / 100:SUM =  INT (SUM * 100 + .5) / 100
450  PRINT K; TAB( 5);R; TAB( 13);SUM; TAB( 22);PD; TAB( 32);NP
460 P = NP
470  NEXT K
480  PRINT : PRINT "MONTHLY PAYMENT IS ";X
490  PRINT : PRINT "TOTAL INTEREST PAID IS ";SUM
500  PRINT : PRINT : PRINT "WOULD YOU LIKE TO RUN AGAIN?";: GET A$
510  IF A$ = "Y" THEN 100
520  IF A$ <  > "N" THEN 500
530  END
```