

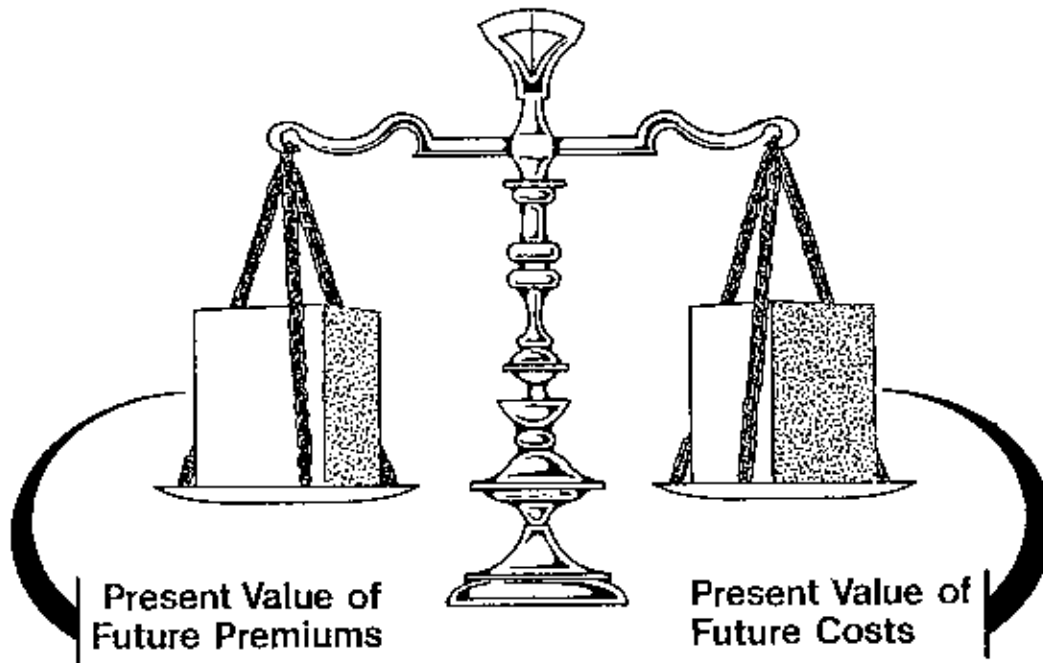
Pricing Auto Insurance

Student Resource Book

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Fundamental Equation of Value





A **premium** is the consideration paid for a contract of insurance for a particular time period.

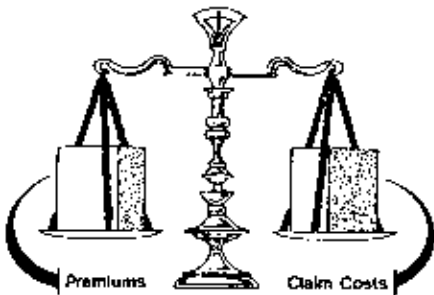
The insured agrees to pay a price, the premium, and the insurance company agrees to pay certain costs for the insured in case of accident. How often the premiums are paid depends on the agreement. Some people like to pay once a year (annually). Some find it easier to pay smaller amounts more often -- quarterly (four times a year) or even monthly. Whatever the frequency, the premium is paid at the beginning of the particular time period to which the premium applies.



The premiums make up the money received by the company for carrying out the insurance contract. This contract is set out in detail in a legal document called the **policy**.

What is a claim? A **claim** is "a demand for something due or believed to be due." In the event that an accident occurs to a policyholder of an insurance company, the policyholder must report the event to the company in order to receive the benefits of the policy. This is known as a claim. The fact that the accident occurred must be verified and an estimate of the resulting damage obtained. Once this is done, the company must pay the policyholder part or all of the damage cost -- whatever has been agreed upon in the policy. Payment of claims is the major cost to the insurance company in dealing with auto insurance.

FIGURE 1
Balancing Income and Costs



Claim cost is the amount paid out in settling claims.

Successful operation of any company is a matter of **balancing** income and costs. Certainly one does not want to pay out more than is taken in. On the other hand, if prices were so high that a company was taking in much more than its expenses, it would be difficult for the company to stay in business because customers would go to other companies where they could get a lower price. In order to arrive at a fair price, the company makes a careful study of its income and its costs. The study of an insurance company's costs is continued in this background information.



Top 2. The company has the following data on accidents:

	Number of Drivers	Number of Accidents per Year
Males	500	30
Females	500	20
Total	1000	50

Total claim costs for all 50 accidents: \$250,000.

- a) What is the average cost per claim?
- b) For each of the three following groups find the annual premium required to cover claim costs:
 - (i) all drivers
 - (ii) male drivers only
 - (iii) female drivers only

Examine the statistics given and the premiums obtained in TOP 2. Should the premium be the same for everyone? Work the following TOP and think about categories based on age as well as sex.

TOP 3. The company has the following data on accidents:

	Number of Drivers	Number of Accidents per Year
Males under age 30	100	10
Males age 30 and over	400	20
Females under age 30	100	5
Females age 30 and over	400	15
Total	1000	50

Total cost of all accidents: \$250,000.

Calculate the annual premium required just to cover the claim costs for each of the four groups of drivers.

C. Additional Costs

The premium charged by an insurance company must cover not only claim costs but also the expenses associated with operating the company.

D. Risk Charge

As we have seen, insurance companies collect a great deal of data, analyze it, and make every effort to be precise in their calculation of costs and expenses from which the premiums are calculated. At best these calculations cannot be more than excellent estimates, since the calculations must be based on past experience. Suppose, for example, the new year brings an unusually high number of claims, or the payments required per claim are unusually high. The estimate for claims payments would then be lower than the actual payments. Expenses might turn out to be much higher than predicted. These and other unforeseen circumstances represent a risk for the insurance company which cannot be avoided. In order to allow for such things, the company adds to the costs an item called **Risk Charge**, which is usually a percentage of the premiums collected.

If the predictions made by the company turn out to be just right, the risk charge represents profit which the company can pass on to its stockholders or policyholders.

If the predictions are too low, the amount of costs over and above the amount predicted must come out of the money included in risk charge. The risk charge would cover the extra cost and prevent a loss to the company.

There is also a possibility that the predictions about cost might be too high. In such a case, the company's profit, includes the risk charge and also some leftover money from the estimate of costs.

The ideal situation, of course, is for the estimates to be as close as possible to what actually happens so that the company charges no more and no less than is needed to keep the balance between income and costs.

Recall:

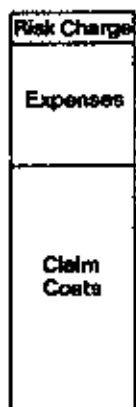
"Percent", denoted %, means "per hundred". Thus 8% is 8/100. For calculation this is best written in **decimal form**, 0.08. A certain percent of a number is the percent times the number:

$$30\% \text{ of } 200 = 0.30 \times 200 = 60.$$

$$30\% \text{ of } P = 0.30 \times P = (0.30)P.$$

FIGURE 3
■■■■■

Total Costs



TOP 5

- (a) Write 65% in decimal form.
- (b) Write 1% in decimal form.
- (c) What is 45% of 60?
- (d) Write 7% of A.

TOP 6

Data:	Number of drivers insured	1000
	Number of accidents per year	50
	Average claim cost	
	per accident	\$5000
	Expenses per policy	\$50
	Risk charge 5% of premium	

Calculate the monthly premium for each driver.

The Balance Principle, Total Income = Total Costs, is still the key to finding the monthly premium in TOP 6, but now we have a better estimate of total costs. If you wish you can now do the preliminary problem, Section V. In the rest of this section we think even more carefully about Income and Costs.



E. Interest

So far we have considered premiums paid as the only source of income for the insurance company. These premiums are paid at the beginning of the pay period (year, quarter, or month.) A responsible insurance company invests the money received from premiums so that it earns interest. For a large company this may be a considerable amount of money. Such a company passes on to the buyer the results of this added income. Before we can calculate the effect this has on the amount of the premium, we review some of the financial concepts involved.

Interest is sometimes called "rent" on money, since it is the amount we pay to borrow money for a period of time. It is also the "rent" a bank pays us for the use of our money when we deposit it in a savings account. The amount we pay (or receive) in interest depends upon the interest rate, the amount we borrow (or deposit), and the amount of time before we pay it back (or withdraw it). Interest is usually stated as a percent for a given period of time, for example, 10% per year. The interest paid on a bank account of \$500 at 10% per year would be at the end of one year:

$$(10/100)(500) = (0.10)(500) = \$50.$$



In this problem we will assume that percents are written as decimals. The statement "the interest rate is I" means that I is written as a decimal. Thus if interest is at 10% per year we would write $I = 0.10$.

The amount on which interest is calculated is called the **Principal**. If the principal is \$500, and the interest paid is 10% per year, the interest at the end of one year is \$50. If the principal is \$10,000, at the same rate of interest, the interest at the end of one year amounts to:

$$(0.10)(10,000) = \$1000.$$

Interest can be calculated for periods other than a year. In six months, the \$10,000 principal at 10% per year would have earned

$$(0.10)(10,000)(6/12) = \$500.$$

In one month the same principal at 10% per year would earn

$$(0.10)(10,000)(1/12) = \$83.33.$$

Simple Interest is interest computed entirely on the principal and is calculated by taking the product:

$$(\text{interest rate per year})(\text{principal})(\text{time period in years}).$$

Suppose you have a savings account in which the money deposited earns 9% per year. You, or perhaps your parents or even your grandparents, may have started this account in order to accumulate enough money to help pay your college expenses. Certainly you do not want to take money out of the account unless it is absolutely necessary, and you want to leave your interest in the account as well. Banks usually calculate the interest on such accounts quarterly, that is, every three months. At the end of the first three months the interest is left in the account and added to the principal. In the next period you receive interest on the new total. The process of collecting interest on the interest earned is called "compounding the interest". **Compound Interest** is simple interest applied over and over to a sum increased by the simple interest earned in each time period.

Example: Suppose you have \$1000 in your account on January 1. The interest at 9% per year in the first three months amounts to $(0.09)(1000)(3/12) = \$22.50$. When your bank statement arrives on April 1 you find your balance is \$1022.50.

Your interest for the next three months is:
 $(0.09)(1022.50)(3/12) = \23.00625 .

On July 1, your balance is
 $\$1022.50 + 23.01 = \1045.51 .

On October 1, your balance is
 $\$1045.51 + (0.09)(1045.51)(3/12) = \1069.03 .

On January 1 of the following year, your balance is
 $\$1069.03 + (0.09)(1069.03)(3/12) = \1093.08 .

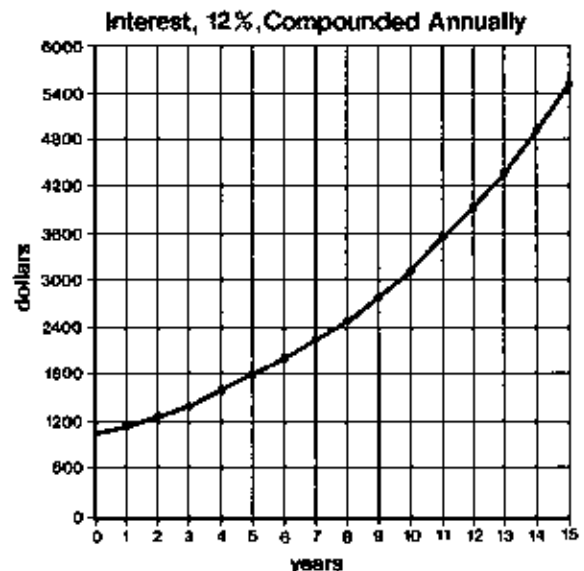
In this example we have been "compounding the interest quarterly". Is this better for you than simply getting 9% for the year? Of course it is. If you got 9% for the year, your account would be \$1090.00 after one year. This may not seem like much difference, but the larger your balance, or the longer the time, the larger the difference will be. For an insurance company it amounts to a great deal of money.

Example:

Suppose, for example, \$1000 is invested for 15 years at 12% per year compounded annually. The following table shows how it grows:

Year	Amount
0	\$1000
1	$(1000)(1.12) = \$1120.00$
2	$(1000)(1.12)^2 = \$1254.40$
3	$(1000)(1.12)^3 = \$1404.93$
4	$(1000)(1.12)^4 = \$1573.52$
5	$(1000)(1.12)^5 = \$1762.34$
6	$(1000)(1.12)^6 = \$1973.82$
7	$(1000)(1.12)^7 = \$2210.68$
8	$(1000)(1.12)^8 = \$2475.96$
9	$(1000)(1.12)^9 = \$2773.08$
10	$(1000)(1.12)^{10} = \$3105.85$
11	$(1000)(1.12)^{11} = \$3478.55$
12	$(1000)(1.12)^{12} = \$3895.98$
13	$(1000)(1.12)^{13} = \$4363.49$
14	$(1000)(1.12)^{14} = \$4887.11$
15	$(1000)(1.12)^{15} = \$5473.57$

FIGURE 4
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Notice that in six years it has almost doubled in value but in 15 years it has increased to almost 5 1/2 times its original value!

Now think about a **general case** and try to work out a formula for calculating compound interest:

Step 1.	Example	General Case
Interest	9%/quarter = 0.0225	I
Principal	\$1000	\$A
Interest (first quarter)	(0.0225)(1000) = \$2.25	\$IA
Step 2.		
Balance	1000 + (0.0225)(1000) = (1000)(1.0225)	A + IA = A(1 + I)
(We used the Distributive Property to simplify.)		
Step 3.		
Interest earned	(0.0225)(1000)(1.0225)	IA(1 + I)
New Balance	(1000)(1.0225) + (0.0225)(1000)(1.0225) = (1000)(1.0225) (1 + 0.0225) = (1000)(1.0225) ²	A(1 + I) + IA(1 + I) = A(1 + I)(1 + I) = A(1 + I) ²

Notice how much easier it is to do this with letters in place of numbers!

TOP 7

Carry on the general case:

- Show that after 3 compounding periods the balance is $A(1+I)^3$.
- After 4 compounding periods what is the balance?

The work you have done leads to the following general statement:

The value of \$A, invested at I per period, at the end of n periods, is \$B, where

$$B = A(1 + I)^n.$$

We often use the term **Future Value** to mean the amount A plus the interest it has earned. The

future value of A, invested at I per period, at the end of n periods is

$$B = A(1 + I)^n.$$



TOP 8

Suppose you have a part-time job and find that you can save \$50 a month. The bank in which you keep your savings compounds interest monthly. The interest earned is 9% per year.

- a) What is I per month?
- b) If you deposit \$50 on January 1, what is its future value on September 1?
- c) You deposit \$50 on February 1. How many months has it earned interest by September 1? What is its future value on September 1?
- d) You deposit \$50 on March 1. What is its value September 1?
- e) Suppose you continue depositing \$50 at the first of the month each month. How much is there in your bank account on September 1? Write your answer as a sum, and then add it up.

F. Present Value

Now look at this matter of interest in a different way. Suppose you want to buy a VCR in 3 months. It will cost \$400. Your bank pays 9% interest compounded monthly. How much must you put in your account now so that you will have \$400 in three months? Like good math students we call the unknown amount \$A. How much is the future value of \$A in 3 months? According to the preceding section:

$$I = (0.09)(1/12) = 0.0075.$$

Future value of \$A in 3 months = $A(1.0075)^3$. We want to choose A so that

$$A(1.0075)^3 = 400$$

$$A = \frac{400}{(1.0075)^3} = \$391.13.$$

The amount we need now to get \$400 three months from now is called the present value of \$400.

The value today of a payment to be made in the future is called the present value of that

payment. (PV is the abbreviation for present value.)

Suppose B is the payment and A is the present value of B at interest rate I for n compounding periods. Then A and B are related as follows:

$$A(1 + I)^n = B, \text{ that is, } A = \frac{B}{(1 + I)^n} .$$

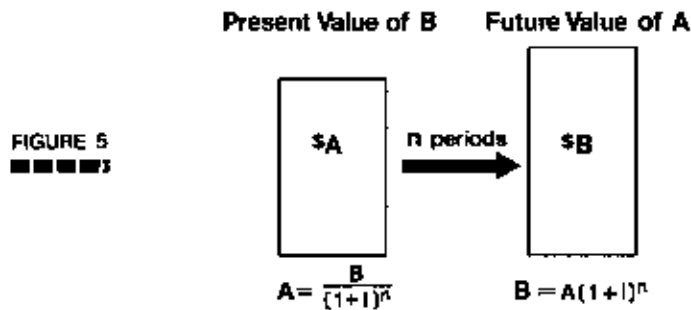


FIGURE 5
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It is handy to write v in place of $\frac{1}{(1 + I)}$. In this notation

$$A = Bv^n .$$

Example: Find the present value of \$1 payable one year from now at 10% interest per year, compounded quarterly.

$$I = 0.025; n = 4; B = \$1; v = 1/1.025 = 0.9756$$

$$A = Bv^4 = \$1 (0.9756)^4 = \$0.91$$

The present value of \$1 is \$0.91, or $PV(1) = 0.91$.

TOP 9

Find the present value of \$100 payable 12 months from now at 1% interest per month.



The present value of a series of payments is the sum of the present values of each of the individual payments. Suppose a payment of \$20 is made on the first of each month, with current monthly interest rate of 0.5%. The present value of the first three payments would be found as follows:

$$v = 1/(1.005)$$

First payment: ($n = 0$)
 $PV(\text{first payment}) = \$20.$

Second payment: ($n = 1$), $A = (20)(v) = \$19.90,$
 $PV(\text{second payment}) = \$19.90.$

Third payment: ($n = 2$), $A = (20)(v^2) = \$19.80,$
 $PV(\text{third payment}) = \$19.80.$

The present value of the first three payments is $20 + 19.90 + 19.80 = \$59.70.$

TOP 10 In each case, find the present value of the series of payments

- (a) \$1, payable at the beginning of each month for one year, with interest at 1% per month.
- (b) \$50, payable at the beginning of each month for one year, with interest at 1% per month.
- (c) \$C, payable at the beginning of each month for one year, with interest rate I per month. (You will need to write your answer as a sum.)

G. The Geometric Series

If you want to, you can keep on writing these long sums. However it is easy to get a little help from your Algebra!

On various national tests you have met questions like this: What do you think the next number should be in the sequence 3, 6, 12, 24, ... ? I am sure you said 48 because you saw that each number is the preceding number multiplied by 2. We could write the sequence as

$$3, 3(2), 3(2)^2, 3(2)^3, 3(2)^4.$$

(What would the next term be? What would the 10th term be?)

A sequence like this is called a **Geometric Sequence**, and if we add these terms together we get a **Geometric Series**:

$$3 + 3(2) + 3(2)^2 + 3(2)^3 + 3(2)^4.$$

We could go on as far as we want. In general we use "a" to represent the first term, and "r" for the number we multiply each term by to get the next one. (In the example $a = 3$ and $r = 2$.) When we add n terms we get

$$S_n = a + ar + ar^2 + \dots + ar^{n-1}.$$

We need a short form for S_n . Look at the first 4 terms, S_4 :

$$S_4 = a + ar + ar^2 + ar^3.$$

Now calculate rS_4 . We get

$$rS_4 = ar + ar^2 + ar^3 + ar^4.$$

Notice that S_4 and rS_4 are quite a bit alike.

$$S_4 - rS_4 = a - ar^4$$

$$(1 - r)S_4 = a(1 - r^4)$$

$$S_4 = \frac{a(1 - r^4)}{1 - r}.$$

Is it always possible to divide by $(1 - r)$? Not if $r = 1$.

Now think about the general case:

$$S_n = a + ar + ar^2 + \dots + ar^{n-1}$$

$$rS_n = ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n.$$

Subtract rS_n from S_n . All the terms cancel except a and ar^n . We are left with

$$S_n - rS_n = a - ar^n,$$

$$(1 - r)S_n = a(1 - r^n), \text{ using the Distributive Property.}$$



If $r \neq 1$, we get

$$S_n = \frac{a(1 - r^n)}{1 - r}.$$

Example:

Use this formula to add $3 + 6 + 12 + 24 + 48$. Here $a = 3$, $r = 2$, $n = 5$. According to the formula,

$$\begin{aligned} S_5 &= \frac{3(1 - 2^5)}{1 - 2} \\ &= \frac{3(-31)}{(-1)} \\ &= 93. \end{aligned}$$

The present value of three \$20 payments on the first of each month with interest 0.5% per month was found to be

$$20 + 20v + 20v^2, \text{ where } v = 1/1.005 = 0.995.$$

This is a geometric series with $n = 3$, $a = 20$, $r = v = 0.995$.

$$S_3 = \frac{20(1 - 0.995^3)}{1 - 0.995} = \frac{20(0.149)}{0.005} = \$59.70.$$

TOP 11

Use the sum of a geometric series to work TOP 10.

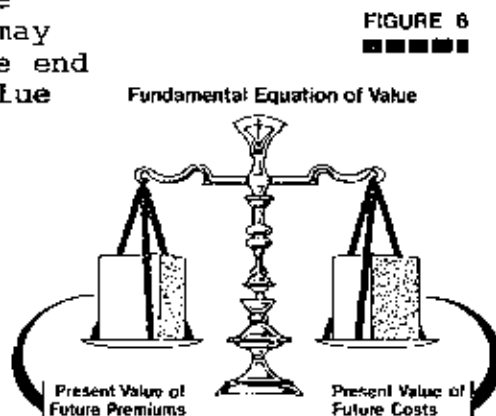
H. The Fundamental Equation of Value

Now we are ready to come back to Pricing Auto Insurance. The idea of balance introduced in Section A is still the basic principle involved, but the items on each side of the balance are a bit more complicated.

We hear frequently in the news media about cheating and stealing on the part of misguided individuals, power-hungry government officials, and even irresponsible executives who set policies for companies. Bad as these cases are, they represent an extremely small percentage of the people in our country. The vast majority of us are like the TV commercial that says "I work an honest day and I want an honest deal."

An honest deal is exactly what the insurance company is trying to provide when it takes into account not only the amounts paid out and received but also the time of year when each of the receipts and payments occurs. Since the decision about the amount of premium must be made at the first of the year, it is natural to consider the value of each transaction at the first of the year, even though such payments or receipts may occur monthly, throughout the year, or at the end of the year. The Fundamental Equation of Value says:

**Present Value of Future Premiums
is equal to
Present Value of Future Costs.**



Remember that costs include claim payments, expenses, and risk charge.

In abbreviated form the Fundamental Equation of Value is **PVFP = PVFC.**

You are now ready to work the problems. The preliminary problem disregards the concept of present value. The problem includes it. Which do you think will give a smaller premium? The challenge problem makes it possible for you to

write a computer program that you can use to answer many interesting questions about premiums.

III. The General Problem

What premium should an insurance company charge for automobile liability and property damage insurance?

You will realize that this general problem is much too complicated to answer completely here. The hope is that you will understand some of the fundamental questions involved and the general principle of balancing income and costs in a very basic situation. When you purchase your own auto insurance you will find many other things must be considered. For example: How do you use your automobile? How far do you drive to work? What part of the country do you live in? What is your accident record? What is the age and sex of each of the drivers? What is the upper limit on claims? Have you had a certified Driver Education Course?

When you buy auto insurance you should consider carefully the reputation of the company you are working with, the restrictions on payment of claims, the record of the company in settling claims, the type of benefits offered, etc. The cheapest is not always the best. Do plenty of reading and research before entering into this contract. Meanwhile here are the problems that American Fidelity wishes you to analyze for them.

IV. The Data

The following data has been obtained from information collected.

Age	Annual Accident Rate per 1000 Drivers	
	Male	Female
Under 20	132	72
20 - 24	95	50
25 - 29	73	38
30 - 34	65	36
35 - 39	53	41
40 - 44	51	33
45 - 49	50	36
50 - 54	47	25
55 - 59	47	27
60 - 64	38	30
65 - 69	42	19
70 - 74	47	32
75 & Over	48	35

Average Cost of Each Accident: \$5,000.



C. Challenge Problem

Create a general formula for calculating a premium using the following notation:

NA = Number of accidents/1000 in group considered

CA = Average claim cost/accident

I = Interest rate per year in decimal form

N = Number of pay periods per year

E = Expenses per policy for commissions, taxes, and fees (% of premium in decimal form)

EI = Cost of issuing policy at the beginning of the year

EP = Cost of premium billing and management/policy at the beginning of each pay period

EC = Cost of claim paying/claim

RR = Risk charge (% of premium in decimal form)

$$v = \frac{1}{1 + \frac{I}{N}}$$

Assume premiums are paid at the beginning of each pay period and claims are paid at the end of the year.

Test your general formula by using the data in part B, The Problem.

