Budgeting Time and Money

Teacher Resource Book

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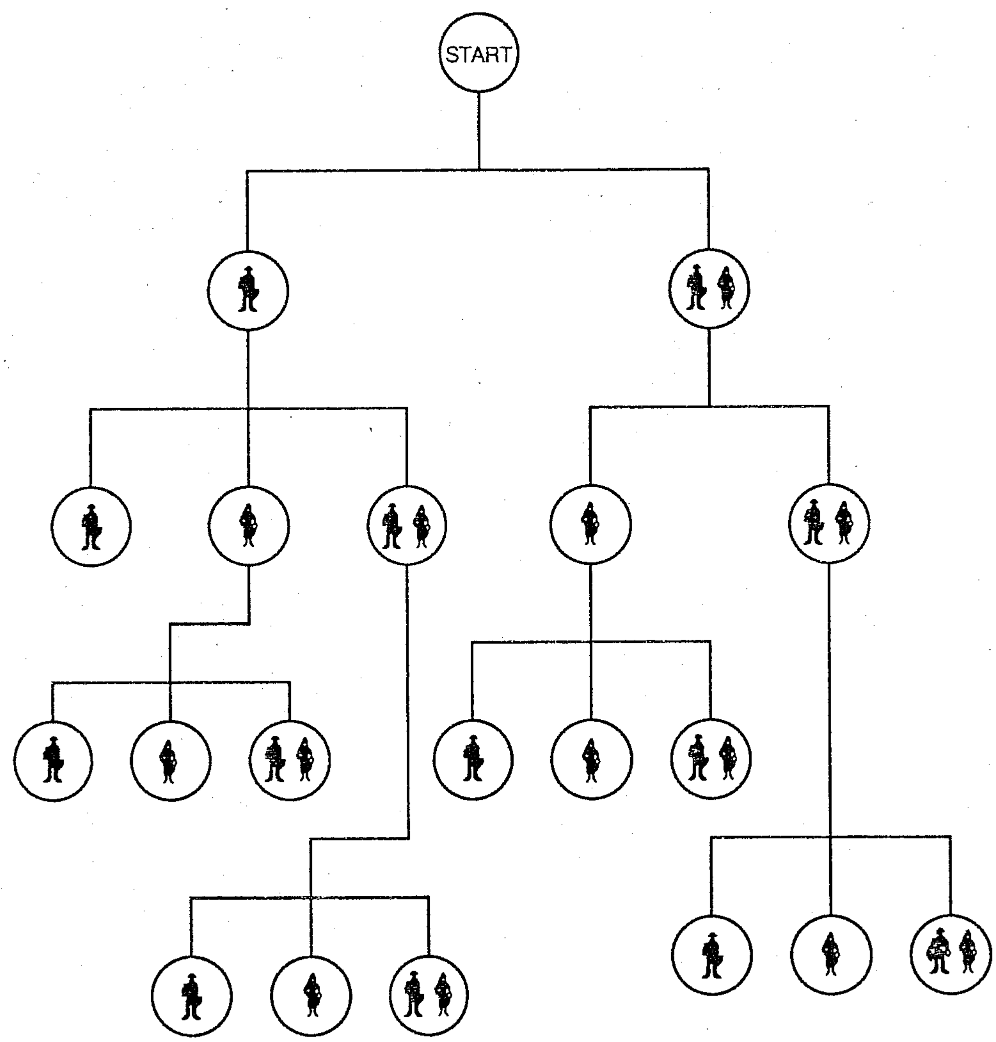

This book is one part of a complete learning module for the problem "Budgeting Time and Money." The entire learning module consists of three video cassettes entitled "The Problem," Problem Preparation," and "A Solution"; a student resource book; a teacher resource book; and a microcomputer diskette for use on an Apple computer.

The book was prepared, published, and distributed by the Mathematical Association of America under the National Science Foundation Grant Number MDR-8470469, entitled "Applications in Mathematics" (AIM).

The materials produced under the AIM project are based on industry-related applied mathematics problems. They have been designed and produced to offer high school teachers a strategy for providing their students an experience in using their reading, writing, and mathematical abilities to solve real problems. In this way the students may realize that these skills are interrelated and that the mastery of them is of vital importance in their future career opportunities.

For more information about this and other AIM modules, write to:

AIM: The Mathematical Association of America
1529 Eighteenth Street, NW
Washington, D.C. 20036

Title:	Budgeting Time and Money
Source:	Arlene M. J. Taylor Information Systems Staff Member AT&T National Product Training Center, Dublin, Ohio
Prerequisites:	No specific prerequisite courses
Skills Needed:	Arithmetic, logic, ability to sort and analyze information
Summary:	The students are asked to devise a scheme for assigning workers to the development of a four-part training program in such a way that the work can be completed in a given length of time at the least cost. They are then asked to alter the development program under certain restrictions so as to complete the development within a given cost limit.
Comments:	Necessary background information is provided. The needed arithmetic can be performed by a calculator. Students gain experience in organization, analysis of given information, making logical decisions, and justifying them. This problem is an excellent vehicle to help students develop problem-solving skills.
Suggested Classroom Uses:	A unit in any high school mathematics or computer course Independent study A project for a math club Enrichment in business mathematics

BUDGETING TIME AND MONEY

Teacher Resource Book

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This project was supported, in part, by the National Science Foundation

Opinions expressed are those of the authors and not necessarily those of the Foundation

I. Introduction

This book is the Teacher Resource Book for the industry-related problem, "Budgeting Time and Money." It is one part of a coordinated package of materials called an AIM Learning Module. The total AIM module consists of the following: (1) Video I, "The Problem"; (2) Video II, "Problem Preparation"; (3) the Student Resource Book; (4) the Teacher Resource Book; (5) one microcomputer BASIC diskette; and (6) Video III, "A Solution."

All three videos are on-site conversations with the industrial representative who has actually worked with this problem. In Video I the discussion centers on the problem and what it means to the company as a whole. In Video II the industrial representative provides some hints to assist the student in making progress toward a solution. In Video III the industrial representative provides her solution of the problem.

There are no specific prerequisite courses for this problem. It involves basic arithmetic, logical thinking, and decision making. The students are required to analyze a situation and make choices. It may surprise them that more than one answer is acceptable.

Section II of the Student Resource Book presents background information about the AT&T National Product Training Center, Dublin, and the concept of a training program. In Section II the particular training program involved is discussed. ***Try Out Problems*** (TOPs) are provided to familiarize students with the concepts at each stage. The general problem is stated in Section IV. Section V-A includes preliminary problems that give the students experience in a variety of situations. This experience will be useful in the problem. The problem is stated in Section IV-B. This is the problem discussed in Videos I and II.

Section II of the Teacher Resource Book, entitled "Teaching Strategies," describes a variety of situations where an AIM learning module can be used. Section III discusses ways in which the teacher might introduce this problem and points out some activities that might help the students get started. Answers to TOPs are included here.

Section IV, entitled "Developmental Approach," sets forth the thought processes through which a student might proceed in reaching a solution to the problem. Since one way of formalizing your thinking is to ask yourself a series of questions, the developmental approach is written completely in the form of questions the students might ask themselves. This section can be used by the teacher as a source of leading questions to guide student discussion of the problem in a class; or it can be given to the students to help them in thinking. If the problem is to be used as independent study, this section will be especially useful. It is probable that for the student's first experience with an AIM module the developmental approach will be important. After the student has worked through one or two of the AIM modules, the student may have acquired enough problem-solving techniques to make the developmental approach unnecessary.

Section V, "A Solution in Detail," gives a complete solution to each problem and makes some comments on questions that might arise.

Section VI discusses the computer program provided. In Section VII the writing of a report is discussed. Section VIII is a collection of additional questions.

II. Teaching Strategies

There are many exciting ways in which the AIM materials can be used in the high school curriculum. You are, of course, free to use your creativity to modify these ways and to devise others that fit your individual situation.

Whatever method you choose, the goal is twofold: (1) to have the students experience mathematics in an industrial setting and (2) to raise the students' awareness of careers in mathematics.

A. A Unit in a Mathematics Course

The objective is to involve the student in a discovery-learning approach geared to developing and sharpening the following skills:

1. reasoning and model building
2. real-world problem solving
3. communicating verbally about mathematics
4. writing technical material concisely and accurately
5. making use of resource materials
6. using the computer in problem solving

Step 1. Preparation. Give each student a copy of the Student Resource Book. (Permission is granted to copy these books for classroom use.) The student reads it and becomes acquainted with the problem and its setting.

Step 2. Video I. The students view Video I. Through this on-site video, they meet the industrial representative who discusses the problem and some related material.

Step 3. Getting Started. The students study the problem, become familiar with the terms, and consult reference books as needed. They talk about the problem--its setting, the technical terms, the assumptions made, and ways in which they might attack it. At some point the students may feel the need for reinforcement and extra direction. They then view Video II, where hints are given by the industrial representative to help them solve the problem.

Step 4. Creating a Solution. The students work as a class or as individuals and discuss their work. At this stage you, as teacher, skillfully nudge the students toward a solution. To help you accomplish this objective, Section IV of this book, entitled "Developmental Approach," provides a succession of questions that you might use to stimulate discussion. A complete solution of the problem is provided in Section V. However, the teacher's role is not to provide a solution, but rather to encourage and tease the students to find their own solution. Students interact with their peers and their instructor and also use the resources of their campus (library, computer center, faculty, etc.). Students come to class ready to report on their mathematical progress. When solutions are obtained, they present their solutions and field questions on their work.

Step 5. The Computation. A partial solution program is provided on a diskette for use on an Apple II. Analysis by hand, along with a hand calculator, is faster and easier than a computer program for this problem.

Step 6. The Report. Each student writes a technical report on the problem and its solution. The report is discussed in Section VII, and a suggested format is given in Section IX.

Step 7. Video III. The students view Video III, "A Solution." At this time they can compare their solution with the one provided on the video.

B. A Unit for Independent Study

The method described in Section A is an "ideal" way to use the AIM materials in a class-oriented problem-solving situation. The same general method is equally effective when used as individual instruction or independent study for one or more students. In such a case, class discussion gives way to periodic teacher conferences. If the student is short of time or unable to proceed, the teacher can provide the list of questions given in the developmental approach (Section IV) to lead the thought processes of the student.

C. Enrichment in a Variety of Courses

Use AIM Videos I, II, and III as a lecture presentation to a class when you wish to stimulate interest in mathematics by demonstrating an application of the material they are studying.

Assign parts of the problem when the classwork deals with some skill used in the problem solution. In this case you might show AIM Video I to acquaint the class with the problem setting and then give a brief discussion of the method of solution.

D. Project for a Math Club

The AIM materials make an exciting series of programs for a math club. The technique described in Section A can easily be adapted to this setting. The fact that math clubs include students at various levels adds interest in the sharing of skills.

E. Developing Career Awareness

All three videos give a firsthand picture of an industrial mathematician at work. Implicitly all three videos raise the students' consciousness of the importance of mathematics in their future career choices. In Video II several persons employed at the AT&T National Training Center, Dublin, discuss their careers and the ways in which the company helps them.

F. Group Presentations

Use the AIM Video as the basis of a presentation at a regional meeting of NCTM.

Use AIM videos and written materials as a resource for a workshop for your area group of mathematics teachers.

III. Where to Begin; Answers to TOPs

A major hurdle in problem solving is deciding where to begin. The first formal step is to read through the Student Resource Book. The students should then view Video I. Some teachers feel it is better to view Video I first and then read the Student Resource Book. Whichever order you choose, it is important to do both since each reinforces the other. Certainly the Student Resource Book cannot be assimilated in a single reading and should be consulted frequently during the problem-solving process.

Encourage students to work each TOP. These problems are designed to help the students assimilate each idea as they proceed. At some point, after the students have attempted some of the preliminary problems, they may wish to view Video II for a few hints. The main emphasis of Video II, however, is career awareness.

Once the overall picture is understood, the REAL fun begins. The order of procedure is laid out in detail. The primary goal is to provide the students the challenge and the satisfaction of making decisions related to a real situation.

Answers to TOPs

TOP 1. a. Suggestions might include the following: car, VCR, TV, refrigerator, microwave, calculator.

b. A possible answer would include the following: course in driving regulations, driving simulator experience, student driver experience, experience in special problems.

TOP 2. This could have several answers: Part 1, or Parts 1 and 3. If the student has a valid reason for the choice, any answer should be acceptable.

TOP 3. They make the course product-specific. Performance-based is also a possible answer.

TOP 4. a. 240 work hours

b. 80 real-time hours

c. $3 \times 80 = 240$ work hours

TOP 5. a. 2 days + 240 hours + 400 hours + 5 days = 16 + 240 + 400 + 40 = 696 real-time hours

b. no

c. yes

d. $16 + 120 + 200 + 40 = 376$ real-time hours

TOP 6. a. 8 hours/day × 5 days/week × \$5/hour = \$200/week

b. $8 \times 5 \times 20 =$ \$800/week

c. $10 \times 8 \times 5 = 400$ work hours per week
Cost = $400 \times 25 =$ \$10,000 per week

TOP 7. a. Field trips cost \$300/day/person; 1 day = 8 hours; Field trips cost 300/8 = \$37.50/work hour; In-house production cost \$25/work hour; Difference = \$12.50/work hour.

b. Travel, lodging, and meals are included in the cost of field trips.

TOP 8. a.

	Real-Time Hours	Work Hours	Cost
Field Trip	16	16	600
Overview	240	240	6000
Operation & Administration	400	400	10000
Field Trial	40	40	2500
Total	696	696	19100

b.

	Real-Time Hours	Work Hours	Cost
Field Trip	16	32	1200
Overview	120	240	6000
Operation & Administration	200	400	10000
Field Trial	40	80	5000
Total	376	752	22200

c. Real-time hours decrease; production cost increases.

TOP 9. a. Both 1 and 2 are not needed since it is not important to know the particular worker. All that matters is whether one or two persons have been assigned to the part.

b. In the right-hand set we have already assigned both workers to Part 1. For Part 2 a single worker may be assigned. It does not make a difference to the time or cost which worker is used.

In the left-hand set we have begun with a single worker. Now we have to decide whether to use the same worker again, use a different worker, or use both of them.

c. In the left-hand side both workers are assigned to Part 2, and one has already been used in Part 1. On the right-hand side both workers have been used in Part 1, and again assigned to Part 2.

IV. Developmental Approach

A. How Shall I Begin?

What are the parts of the training program?

I need to think about time and cost. What changes the time? What changes cost? How does time change? How does cost change?

I should make a chart so that I can get the whole picture.

I need to include a column for the time needed for one worker to do the entire development. How many columns should I make?

Is the time for two workers half the time for one worker? Why not?

Is the cost for two workers twice the cost for one worker? Why not?

If I make a chart for time and one for cost, I can include more workers on each chart. What would be a good number to start with?

B. Preliminary Problems

A1, A2, A3

I can work problem A1 directly from my chart.

What about A2? Is total cost the same? Why not?

What about real time? Why does this change so much?

If I try A3 and think about what is happening, perhaps I can figure out why things change as they do.

Why does it not make sense to add the times if different workers do different parts?

Really this is rather like a race. The race is done when the last runner finishes. Is the real time the time when the last worker finishes?

A4 and A5

This sounds a bit complicated. Can I possibly try all possible ways to assign workers?

How many ways can I put three people in four chairs? There are $4 \times 3 \times 2 = 24$ ways of doing this. And there must be many more assignment plans. Should I try one and then see how it can be improved?

What would be a good way of writing out the plan so that I can see at a glance what part each worker is assigned to handle and how many workers are on each part?

Is this a balancing act? What happens to the time if I put three workers in place of two on Part 1? But I have only four workers. What happens to the total time if W1 has to work on three parts instead of just one part?

This is rather fun. Once I get a pretty good plan, can I find a better one?

Is A5 easier than A4? Why?

C. The Problem

What appears to be my first task here? The important thing seems to be time.
Can I use the technique I developed in A5 to help me design a plan for this problem?
Do I have a limit on the number of workers?
Do I want the plan that takes the least time?
I am able to get several plans under 600 hours. How can I choose the one of least cost?
Am I sure that this plan is the least expensive one that can be carried out under 600 hours? Why?

The second part adds a limit on cost. Is this cost more than the cost I got for the least expensive plan in the first part?
Can I hope to find a plan that will complete the entire development for $60,000? Why not?
What part of the program should I cut out?
How could I justify eliminating this part?
What would be the least expensive way of doing the new development? Can I use what I learned in the preliminary problems to decide on the least expensive assignment?

Is this the only way I could cut things out to save money?

V. A Solution in Detail

A. Getting Ready to Begin

After the students have done TOP 8, it should be clear that some method of organizing the information about time and cost would be helpful. A suggested collection of useful information is illustrated in Charts 1 and 2. Although the first preliminary problem asks only for the hours and cost with one person throughout, the students will save time, and have valuable information to work with, if they prepare a complete chart based on 1, 2, 3, 4, or 5 workers. It is simpler to have separate charts for time and cost. (See Charts 1 and 2.)

Through preparing the charts, the students discover the way in which the number of workers affects time and cost in different activities.

1. The number of workers ***does not affect the work hours*** involved in creating the courses. The number of workers ***does affect the real time*** involved in creating the courses.

2. The number of workers ***does not affect the real time involved*** in field trials and field trips. The number of workers ***does affect the work hours*** and therefore the cost involved in field trials and field trips. In Part 1, for example, one person on a two-day field trip costs $600, two persons on a two-day

CHART 1

REAL-TIME HOURS

Part	Number of Persons				
	1	2	3	4	5
Part 1:					
Field Trip, 2 days @ $300	16	16	16	16	16
Overview	240	120	80	60	48
Operation and Admin.	400	200	133	100	80
Field Trial	40	40	40	40	40
Total	696	376	269	216	184
Part 2:					
Field Trip	32	32	32	32	32
Installation	800	400	267	200	160
Field Trial	40	40	40	40	40
Total	872	472	339	272	232
Part 3:					
Field Trip	32	32	32	32	32
General Maintenance	240	120	80	60	48
Special Maintenance	640	320	213	160	128
Field Trial	48	48	48	48	48
Total	960	520	373	300	256
Part 4:					
(Field Trip)	(32)	(32)	(32)	(32)	(32)
General Engineering	80	40	27	20	16
Advanced Engineering	240	120	80	60	48
Field Trial	40	40	40	40	40
Total	360 (+32)	200 (+32)	147 (+32)	120 (+32)	104 (+32)

Formulas: Part 1. $56 + \frac{640}{n_1}$ — n_1 = number assigned to Part 1

Part 2. $72 + \frac{800}{n_2}$ — n_2 = number assigned to Part 2

Part 3. $80 + \frac{880}{n_3}$ — n_3 = number assigned to Part 3

Part 4. $40 + \frac{320}{n_4}$ — n_4 = number assigned to Part 4

or

$40 + \frac{320}{n_4} + 32$

Let Wi = workers i = 1, 2, ..., k.
Ti = total numbers of real-time hours by Wi

Total Time = Max $\{T_1, T_2, \ldots, T_k\}$

field trip costs $1200, three persons on a two-day field trip costs $1800, etc. But the field trip requires two days (16 real-time hours) no matter how many go.

A student might use many approaches in attacking the problems assigned. Basically, logic is required and the paths of logic are different for different persons. Students should be encouraged to proceed as they wish but also to present some justification for their approach.

The analysis of cost is simpler than the analysis of time. After preparing the charts, the students should recognize some basic facts about cost:

Part 1. No matter how many persons are involved, Part 1 requires 640 work hours at $25 an hour. Thus the fixed cost in Part 1 is $16,000. Field trips and field trials each depend on the number of

CHART 2

PRODUCTION COST

Part	Number of Persons 1	2	3	4	5
Part 1:					
Field Trip, 2 days @ $300	600	1200	1800	2400	3000
Overview	6000	6000	6000	6000	6000
Operation and Admin.	10000	10000	10000	10000	10000
Field Trial	2500	5000	7500	10000	12500
Total	19100	22200	25300	28400	31500
Part 2:					
Field Trip	1200	2400	3600	4800	6000
Installation	20000	20000	20000	20000	20000
Field Trial	2500	5000	7500	10000	12500
Total	23700	27400	31100	34800	38500
Part 3:					
Field Trip	1200	2400	3600	4800	6000
General Maintenance	6000	6000	6000	6000	6000
Special Maintenance	16000	16000	16000	16000	16000
Field Trial	1200	2400	3600	4800	6000
Total	24400	26800	29200	31600	34000
Part 4:					
(Field Trip)	(1200)	(2400)	(3600)	(4800)	(6000)
General Engineering	2000	2000	2000	2000	2000
Advanced Engineering	6000	6000	6000	6000	6000
Field Trial	2500	5000	7500	10000	12500
Total	10500	13000	15500	18000	20500

Formulas: Part 1. $16000 + n_1(3100)$

Part 2. $20000 + n_2(3700)$

Part 3. $22000 + n_3(2400)$

Part 4. $8000 + n_4(2500) + n^* (1200)$

Total Cost = $66000 + n_1(3100) + n_2(3700) + n_3(2400) + n_4(2500) + n^*(1200)$

n_1 = number assigned to Part 1

n_2 = number assigned to Part 2

n_3 = number assigned to Part 3

n_4 = number assigned to Part 4

n^* = number assigned to Part 4 and not to Part 3

persons involved. Let n_1 be the number working on Part 1. The variable cost is $n_1(600 + 2500) = n_1(3100)$dollars.

$$Cost\ Part\ 1 = \$16,000 + n_1(3,100).$$

Part 2. $$Cost\ Part\ 2 = \$20,000 + n_2(3,700).$$

Part 3. $$Cost\ Part\ 3 = \$22,000 + n_3(2,400).$$

Part 4. $$Cost\ Part\ 4 = \$8,000 + n_4(2,500) + n^*(1,200),$$

where n^* is the number of persons working on Part 4 and not on Part 3. This extra term allows for the fact that the field trip for Part 3 also serves Part 4.

$$TOTAL\ COST = \$66,000 + n_1(3,100) + n_2(3,700) + n_3(2,400) + n_4(2,500) + n^*(1,200).$$

A similar technique can be used to analyze the real time required to perform each part.

Part 1. Fixed time: 7 days or 56 hours for field work.
Variable time: $640/n_1$, where n_1 is the number of persons working on Part 1.

Time Part 1 = 56 + 640/n_1 hours.

Part 2. *Time Part 2 = 72 + 800/n_2 hours.*

Part 3. *Time Part 3 = 80 + 880/n_3 hours.*

Part 4. *Time Part 4 = 40 + 320/n_4 or 40 + 320/n_4 + 32*

if some person working on Part 4 is not working on Part 3.

We are now tempted to add to obtain the total time. However, the total time required for all four parts depends on how the personnel is assigned.

The purpose of the preliminary problems is to give the students an opportunity to explore a variety of situations so they can discover how to assign personnel to reduce time without increasing cost beyond the assigned limits. The students will probably work directly from their charts without using the formulas.

B. Preliminary Problems A1, A2, A3

A1

Preliminary problem A1 provides the simplest possible situation. The total cost is found by adding. The number of hours is found in the same way.

Total cost \$77,700; number of hours 2888.

A2

An extra 1,200 is required because a field trip will have to be made by the person assigned to Part 4.

Total cost \$77,700 + 1,200 = \$78,900.

It is necessary to think carefully in order to find the time. Different persons are working each part. The four parts can be done in any order. The quickest way is to do them at the same time. The total real time involved will be the time it takes to perform the longest part.

Time = max {696, 872, 960, 392} = 960 hours.

A3

Again cost is the easiest to calculate. It is simply the sum of the costs incurred in carrying out each part:

$$Cost = (19{,}100 + 23{,}700) + (24{,}400 + 10{,}500) = \$77{,}700.$$

The first worker is engaged in both Parts 1 and 2. To complete both of these parts, 696 + 872 hours are needed. The second worker must complete both Parts 3 and 4. The time needed is 960 + 360. Since the two can work at the same time, the real time needed is given by:

$$Time = \max \{696 + 872,\ 960 + 360\} = 1568 \text{ hours}.$$

C. Preliminary Problems A4, A5

A4

In Problem A4 we need the least time required to develop the training program if four workers are available. At this point we need to consider a variety of possible assignments. We could, of course, simply try every possibility, but there are a great many!

Since the job is done if and only if each worker assigned to it has finished his assignment, the total time to complete the job is the largest number of hours worked by any worker. If one worker (W1) is assigned to Parts 1 and 3, the total time of work for W1 is the total of the real time needed to do Part 1 plus the real time needed to do Part 3. This real time will, of course, depend on how many other workers are assigned to Part 1 and how many are assigned to Part 3. A table of the following type is one way of seeing the total picture for a given assignment.

Assignment	Part 1	Part 2	Part 3	Part 4	Real Time
W1	*x*				*376*
W2	*x*	*x*	*x*		*376 + 472 + 373*
W3		*x*	*x*		*472 + 373*
W4			*x*	*x*	*373 + 360*

The number 376 comes from the fact that two persons work on Part 1. The number 472 comes from two persons on Part 2. The number 373 is the real time for three persons on Part 3. The number 360 is real time for one person on Part 4 (assuming that person has taken the field trip in Part 3).

Read across the table to find the total time for each worker. The total time to complete the development is the largest time worked by any worker. In the above example it is 1221 hours. Clearly this is not a good assignment since one worker puts in so many more hours than the others.

A better way to assign four workers would be to use different workers on the parts that require most time and thus even out the time for each worker. On the other hand, it is desirable to keep the workers from being involved in too many parts. Although the cost per part decreases, the total cost may increase.

Each of the following plans represents a reasonable plan if time is being considered. Since the problem asks also for the corresponding cost, the cost is included in the chart for convenience.

PLAN A

Assignment	Part 1	Part 2	Part 3	Part 4	Real Time
W1	*x*				*696*
W2		*x*			*872*
W3			*x*		*960*
W4				*x*	*392*

Time = 960 hrs

Cost = 19100 + 23700 + 24400 + 11700 = $78,900

The real time and cost figures come from Charts 1 and 2. Notice that the time for Part 4 includes 32 hours for the extra field trip, and the cost includes $1200 for the same reason

PLAN B

Assignment	Part 1	Part 2	Part 3	Part 4	Real Time
W1	*x*	*x*	*x*	*x*	*908*
W2	*x*	*x*	*x*	*x*	*908*
W3	*x*	*x*	*x*	*x*	*908*
W4	*x*	*x*	*x*	*x*	*908*

Time = 908 hrs

Cost = 28400 + 34800 + 31600 + 18000 = $112,800

Plan A and Plan B represent two "extreme" cases. In Plan A the workers do not overlap. In Plan B each part is done in the least time. Plan B takes less time than Plan A, but Plan B is much more costly. Surprisingly, neither one represents the optimum assignment from the point of view of time.

The Branch and Bound method is described briefly in the student book and in more detail later in this section. This method gives an organized way of thinking about this problem. Whatever method is chosen, a great many possibilities exist. The students should be encouraged to think out a way of approaching the problem, perhaps even create their own algorithm. The class should compare answers and approaches and in this way experience the fun of a give-and-take learning experience. Above all, finding the best plan of assigning the workers is not nearly as important as participating in the search.

After several unproductive approaches, I finally came up with the following thought process:

Part 3 is the longest. Begin by putting two workers on this part and letting the other two do the rest.

PLAN C

Assignment	Part 1	Part 2	Part 3	Part 4	Real Time
W1			*x*		*520*
W2			*x*		*520*
W3		*x*			*872*
W4	*x*			*x*	*696 + 392*

Time = 1088 hrs

Cost = 19100 + 23700 + 26800 + 11700 = $81,300

Plan C is worse than either A or B. W4 works the longest. How about letting W1 and W2 do Part 4 since W1 and W2 work the shortest length of time.

PLAN D

Assignment	Part 1	Part 2	Part 3	Part 4	Real Time
W1			*x*	*x*	*520 + 200*
W2			*x*	*x*	*520 + 200*
W3	*x*				*696*
W4		*x*			*872*

Time = 872 hrs

Cost = 19100 + 23700 + 26800 + 13000 = $83,000

Plan D is the best effort so far, but is it the shortest? At least we have an upper bound on our time. Any plan that takes more than 872 hours must be thrown out. It is tempting to try W1, W2, and W3 on Part 3 and W4 on Part 2. This plan would mean that Part 3 would require 373 hours, and Part 2 would need 872 hours. We cannot use W4 on anything more than Part 2 without exceeding the bound. At best we can only match it, so throw that idea out.

Try instead, putting W1, W2, W3 on Parts 2, 3, and 4, and W4 on Part 1. (The actual workers do not matter. What matters is whether they are repeated in a project.)

PLAN E

Assignment	Part 1	Part 2	Part 3	Part 4	Real Time
W1		*x*	*x*	*x*	*339 + 373 + 147*
W2		*x*	*x*	*x*	*859*
W3		*x*	*x*	*x*	*859*
W4	*x*				*696*

Time = 859 hrs

Cost = 19100 + 31100 + 29200 + 15500 = $94,900

This is the best yet. We now have a new bound, 859 hours. Is there a way to improve this plan? W4 seems to be getting too light a load. The fact that the assignment is so uneven in distribution makes one suspect that there may be a better way.

Encourage the students to try different arrangements. After they have worked this way, they may enjoy the branch and bound method that does give one the satisfaction of having tried all possibilities. The shortest time comes from the following arrangement. (At least this is the shortest time I found. What fun it would be for the students to find that I have overlooked something!)

PLAN F

Assignment	Part 1	Part 2	Part 3	Part 4	Real Time
W1			x	x	520 + 200 = 720
W2			x	x	720
W3	x	x			376 + 472 = 848
W4	x	x			848
					Time = 848 hrs

Cost = 22200 + 27400 + 26800 + 13000 = $89,400

Plan F is symmetric, the length of work time is about even, and the assignment of W1W2 and that of W3W4 are disjoint. Plan G has these properties also. What makes Plan F shorter?

PLAN G

Assignment	Part 1	Part 2	Part 3	Part 4	Real Time
W1	x		x		376 + 520 = 896
W2	x		x		896
W3		x		x	472 + 232 = 704
W4		x		x	704
					Time = 896 hrs

Cost = 22200 + 27400 + 26800 + 15400 = $91,800

A5

Certainly the least cost does not occur when the time is least. For Plans A through G the smallest cost is $78,900. But is this cost actually the least? It is much easier to find the least cost than to find the least time. The formula

$$Cost = 66000 + n_1(3100) + n_2(3700) + n_3(2400) + n_4(2500) + n^*(1200)$$

shows that the cost is least if $n_1 = n_2 = n_3 = n_4 = 1$ and $n^* = 0$. This calculation determines the following assignment chart for least cost:

Assignment	Part 1	Part 2	Part 3	Part 4	Real Time
W1	x				696
W2		x			872
W3			x	x	960 + 360

Time = 1320 hrs

Cost = 19100 + 23700 + 24400 + 10500 = $77,700

This plan uses only three people and requires 1320 hours. The development could also be done for $77,700 by one person for the same cost, but the time would increase to 2888 hours. (See A1.) This chart represents the shortest time for this cost.

D. The Problem

Since CCC requires its training program to be completed in 4 months, our first task is to seek an assignment of workers that can do the entire job in a total of 4 months; that is 1800/3 = 600 hours. We can use as many workers as desired. No worker can work more than 600 hours. A glance at our charts tells us that no worker can work alone on Part 1, Part 2, or Part 3. An obvious way to get the time below 600 hours is to use the following plan:

PLAN B1

Assignment	Part 1	Part 2	Part 3	Part 4	Real Time
W1	x				376
W2	x				376
W3		x			472
W4		x			472
W5			x		520
W6			x		520
W7				x	360 + 32

Time = 520 hrs

Cost = 22200 + 27400 + 26800 + 11700 = $88,100

Since no one can exceed 600 hours, W1 and W2 can work at most 224 hours more. Four persons working on Part 2 require 272 hours each. If W1 and W2 work on Part 1 and are also assigned to Part 2 along with W3 and W4, W1 and W2 will work more than 600 hours. However, W4 and W5 can also work 128 more hours. If the same four workers are assigned to Parts 1 and 2, each worker works 216 + 272 = 488 hours, well within the bound. In fact, if all four work on Part 1 and three of them work on Part 2, these two parts can be done in 216 + 339 = 555 hours.

Continued analysis like this can lead to several ways of assigning work so that the total production is done in 600 hours. Two further plans are listed. There are many more.

PLAN B2

Assignment	Part 1	Part 2	Part 3	Part 4	Real Time
W1	x				216
W2	x	x			269 + 339
W3	x	x			216 + 339
W4	x	x			216 + 339
W5			x	x	373 + 200
W6			x	x	373 + 200
W7			x		373

Time = 573 hrs

Cost = 28400 + 31100 + 29200 + 13000 = $101,700

PLAN B3

Assignment	Part 1	Part 2	Part 3	Part 4	Real Time
W1	x	x	x	x	578
W2	x	x	x	x	578
W3	x	x	x	x	578
W4	x	x	x	x	578
W5	x	x	x	x	578
W6	x	x	x	x	578
W7	x	x	x	x	578
W8	x	x	x	x	578

Time = 578 hrs

Cost = 66000 + 8(3100 + 3700 + 2400 + 2500) = $159,600

Is it clear that assigning all workers to all parts is the most costly procedure?

Plan B1 is the least expensive of the plans we have suggested. If we look at the formula for cost, we can see that Plan B1 must be the least expensive. Consider the following argument:

1. Parts 1, 2, and 3 must each have at least two persons in order to complete the part in 600 hours. Thus n_1, n_2, and n_3 must all be greater than or equal to 2. The least cost choice is 2.

2. The value of n_4 can be as low as 1. If W5 or W6 is assigned to Part 4 in addition to Part 3, the work time for that worker will become 720 unless another worker is assigned. Putting another worker on Part 3 will increase cost more than paying the extra $1200 for W7 to attend the field trip.

Conclusion: The least amount of money required to complete the entire development is $88,100.

Part 2 of Problem B

The upper limit set for cost is $60,000. However, if all parts are developed, fixed cost is already $66,000. There is no way of reducing this cost without eliminating one of the parts or part of one of the parts of the development.

Plan 1. Eliminate Part 2

Assignment	Part 1	Part 3	Part 4	Real Time
W1	*x*			*376*
W2	*x*			*376*
W3		*x*		*520*
W4		*x*		*520*
W5			*x*	*392*

The job can be completed in 4 months.

Cost: 22200 + 26800 + 10500 + 1200 = $60,700

The cost and time are both within the bounds requested by CCC. This plan is reasonable because installation is a separate activity and a one-time activity. It therefore could be eliminated from the general Product Training Program.

The argument against this choice is that the installation would have to be performed by a trained person and would, therefore, be an additional expense to the purchaser of the product. The CCC might prefer to include the instruction in installation free as a promotional activity.

Plan 2. Eliminate One or More Portions of Some Parts

A possible omission would be the Special Maintenance course in Part 3. This would reduce the time for Part 3 by 640 hours in the case of one person, 320 hours in the case of two persons, etc. The work plan in this case might be as follows:

Assignment	Part 1	Part 2	Part 3	Part 4	Real Time
W1	*x*				*376*
W2	*x*				*376*
W3		*x*			*472*
W4		*x*			*472*
W5			*x*		*320*
W6				*x*	*320*

Cost = 22200 + 27400 + 8400 + 10500 + 1200 = $69,700

It is still necessary to pare about $9,000 from this cost. It cannot be taken from Part 4 since this part was assumed to be important. No individual can be taken from Parts 1 or 2 without increasing the time. Some change in the assumptions is needed.

Assume that only one individual must attend field trips and field trials. This reduces the cost of Part 1 by $3100 and the cost of Part 2 by $3700, a total of $6800 reduction. This amount is not enough.

Assume in addition that the field trial for Part 1 is held in-house. The cost of this trial becomes 40 × 25 = $1000, in place of the original $2500.

With these changes in the assumptions, the cost becomes:

Part 1: *600 + 16000 + 1000 = 17600*

Part 2: *1200 + 20000 + 2500 = 23700*

Part 3: *1200 + 6000 + 1200 = 8400*

Part 4: *10500 + 1200 = 11700*

Total: $61,400; almost within the limit requested.

The justification for removing the Special Maintenance course is that that particular material might be covered, in part at least, in the Engineering course.

The justification for cutting down the number of workers attending the field trips and field trials is that a single individual can report to other workers any needed information. The justification for holding the field trial for Part 1 in-house is that it can be done as effectively there and will save considerable cost.

The students may be able to come up with a variety of alternate plans. This problem is a problem that has an answer--not the answer.

E. Branch and Bound

In mathematical terms, the problem here falls into the category of *Optimization*. There is a time constraint given: the time is to be less than 600 hours. The problem asks for the least cost subject to this constraint. One way to organize the solution is called branch and bound. The general idea of this technique is introduced in the Student Resource Book.

Start at time 0. From this point, consider branches representing possible assignments to complete Part 1. For each branch assign the time required. Continue with a further set of branches coming from the endpoints of the first set. With each of these sets, associate the time to complete the first two parts of the total assignment. Continue in this way. At each point in the procedure, keep track of the total time required. As you can guess, this procedure could get very complicated. If a time bound is given, a particular branch can be discarded as soon as the time exceeds this bound. If a bound is given on cost, keep track of the cost so that it can be used to abandon branches that prove too expensive. If no bound is given, it is wise to create a bound by following down a particular line of assignment (choose an easy one) and use this bound to limit the number of branches that must be pursued.

With four parts and a large number of possible workers, this process can become quite complicated when done with a hand calculator. There is, of course, no law against using a reasoning process to cut down on the work. A simple example is given in the Student Resource Book, in which the shortest time is found for two workers to complete Parts 1 and 2.

Before giving the solution to the problems in this format, an additional example is given here.

Example 1

Suppose three workers are assigned to perform Parts 1 and 2. What is the least cost at which the work can be done in less than 800 hours?

Solution:

In order to do Part 1 we can assign W1, W2, W3, or W1W2, W1W3, W2W3, or W1W2W3. How many of these choices are really different? The only important consideration at this stage is not the particular worker, but how many workers are assigned to Part 1. Thus the choices in line 1 of Figure 1 should be W1, W1W2, and W1W2W3. The number in the circle represents the worker; the number beside the circle is the number of real-time hours at the end of Part 1.

Now move to Part 2. In the branch beginning with W1 we can now assign W1 again, a different single worker (the choice between W2 and W3 at this point makes no difference), two workers including W1 (call them W1W2), two workers not including W1 (call them W2W3), or all three workers W1W2W3. Similarly, we follow possible assignments for each of the three branches begun in line 1.

The time bound is 800 hours. We can now use this bound to eliminate several of the possibilities. There are four ways in which the time is less than 800 hours. We must calculate the cost for each of these ways. In this case the least expensive way is not the shortest.

The least cost assignment is W1 on Part 1 ($19,100) and W2W3 on Part 2 ($27,400), making a total cost of $46,500. See Figure 1.

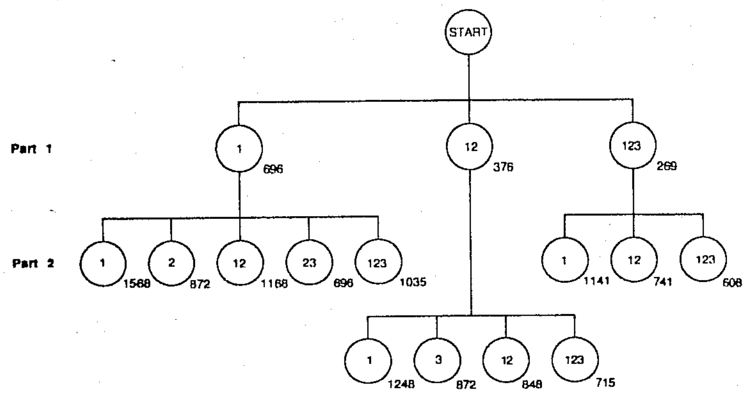


Figure 1

> ***Example 2***
>
> Two workers are assigned to complete Parts 1, 2, and 3. What is the least time in which this assignment can be done and how should the workers be assigned?

Solution:

Since no bound was given, we can create one by using both workers on each of the three parts: 376 + 472 + 520 = 1368. The shortest time is found to be 1336 hours, obtained by assigning W1 to Parts 1 and 3 and W2 to Parts 1 and 2. See Figure 2.

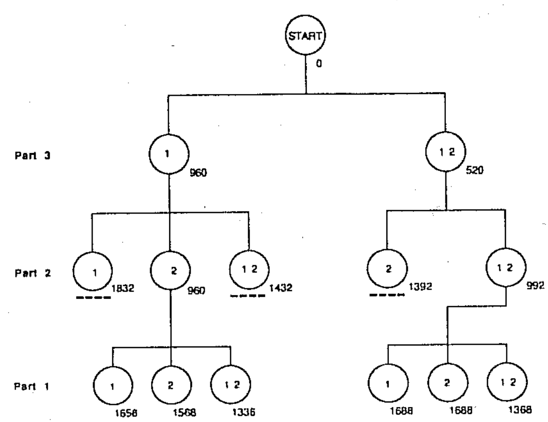


Figure 2

Problems A4 and A5

The branch and bound method is applied to the preliminary problems A4 and A5 and to the problem as shown in the following diagrams. Problem A4 is analyzed in Figure 3. It is helpful to have available an abbreviated version of the time and cost charts as shown.

REAL-TIME HOURS

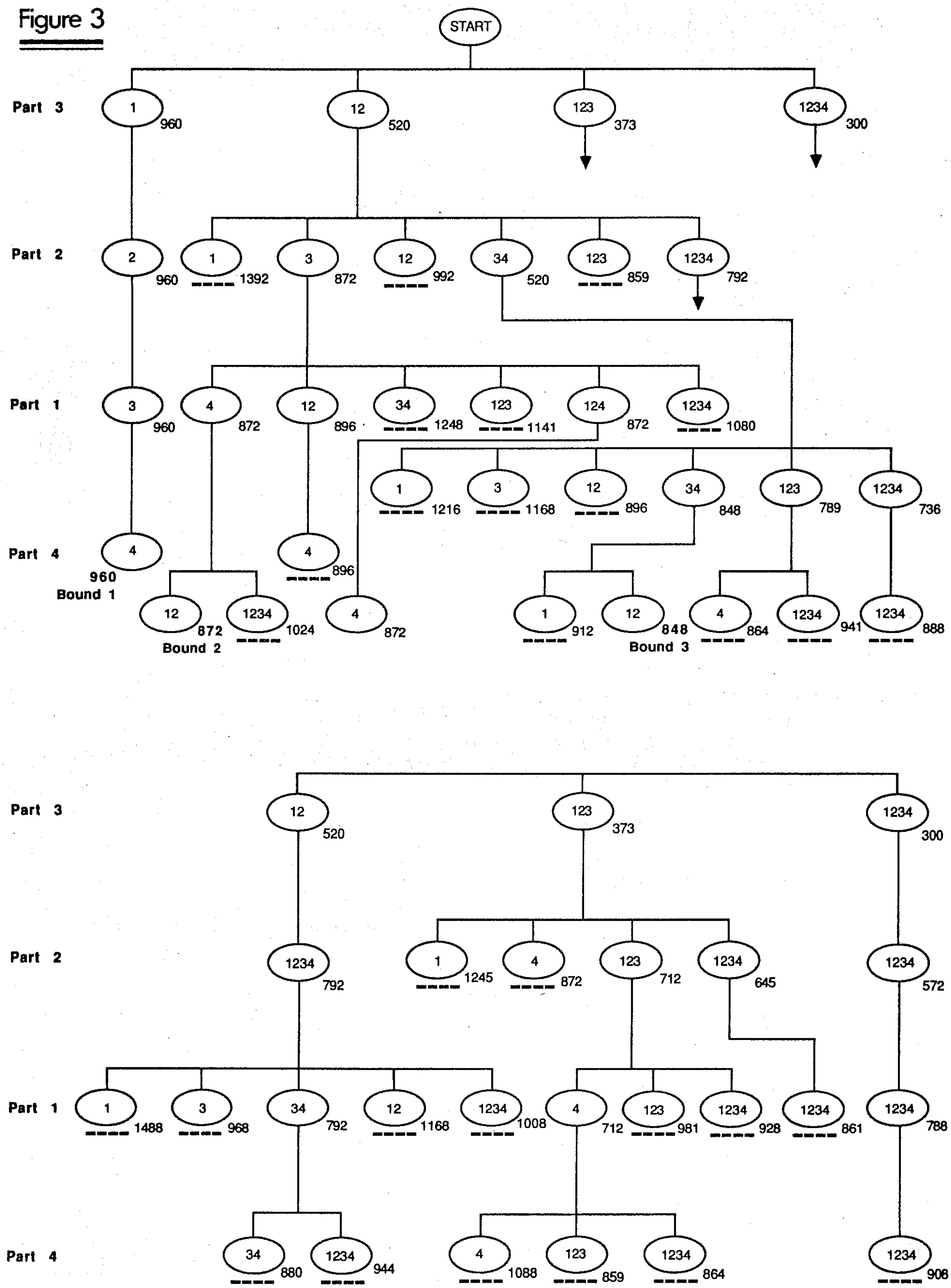
Part	Number of Workers on Part							
	1	2	3	4	5	6	7	8
1	696	376	269	216	184	163	147	136
2	872	472	339	272	232	205	186	172
3	960	520	373	300	256	227	206	190
4	360	200	147	120	104	93	86	80
or	392	232	179	152	136	125	118	112

COST IN HUNDREDS OF DOLLARS

Part	Number of Workers on Part							
	1	2	3	4	5	6	7	8
1	191	222	253	284	315	346	377	408
2	237	274	311	348	385	422	459	496
3	244	268	292	316	340	364	388	412
4	105	130	155	180	205	230	255	280

(Add n*1200 for each worker on Part 4 that is not on Part 3.)

Figure 3


START
Part 3
1 960
12 520
123 373
1234 300
Part 2
2 960
1 1392
3 872
12 992
34 520
123 859
1234 792
Part 1
3 960
4 872
12 896
34 1248
123 1141
124 872
1234 1080
1 1216
3 1168
12 896
34 848
123 789
1234 736
Part 4
4
960
Bound 1
12 872
Bound 2
1234 1024
4 896
4 872
1 912
12 848
Bound 3
4 864
1234 941
1234 888
Part 3
12 520
123 373
1234 300
Part 2
1234 792
1 1245
4 872
123 712
1234 645
1234 572
Part 1
1 1488
3 968
34 792
12 1168
1234 1008
4 712
123 981
1234 928
1234 861
1234 788
Part 4
34 880
1234 944
4 1088
123 859
1234 864
1234 908

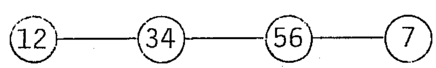
The Problem

Here we are given a time bound, 4 months. In hours this bound means 600 hours. We are not given a specific number of workers. Since we can begin the analysis with any part, it is useful to choose Part 3, which requires the longest time, since this may eliminate choices more rapidly. See Figure 4.

The choice of worker 1 alone on Part 3 makes the time exceed 600 hours, and we abandon this branch immediately. In addition, we can see that a single worker cannot be assigned to Part 1, Part 2, or Part 3 without exceeding the time limit.

Now consider the branch beginning with W1 and W2 on Part 3. Their completion time is 520 hours. We can now see that neither W1 nor W2 can be assigned to Part 2 without exceeding 600 hours. The only possible choices are W3 and W4, along with as many other workers as seems desirable.

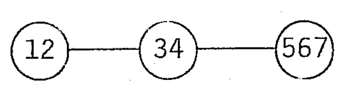
Let us follow the branch



.

At each part new workers are included so that the total time remains the maximum of the times for each part, 520 hours. This process has been accomplished with seven workers. Since the cost increases with each additional worker and since we want the minimum cost, this procedure establishes that we need not consider more than seven workers. Thus we can eliminate a choice using W8, W9, ...

Now look at the branch

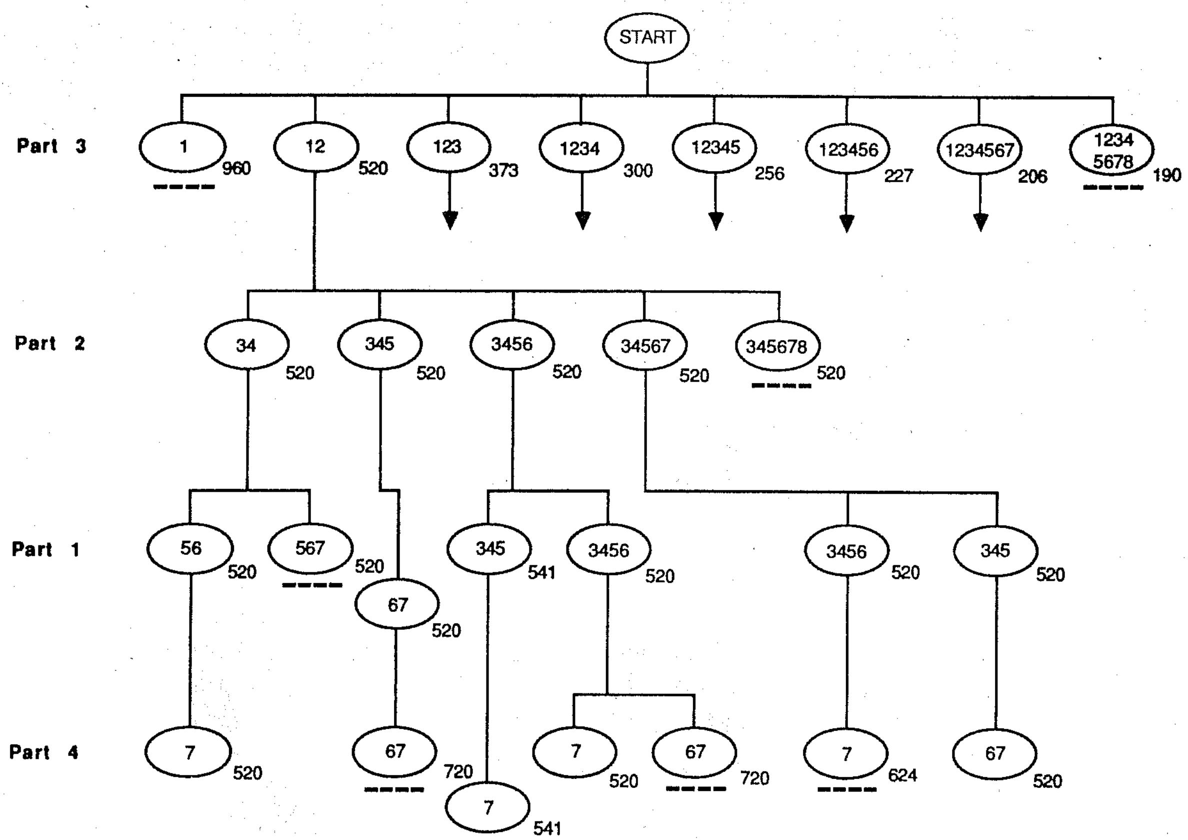


.

This branch must end here since T = 520, and one of the workers must be used again to work on Part 4. This process cannot be done in 80 hours, even using all seven workers on Part 4.

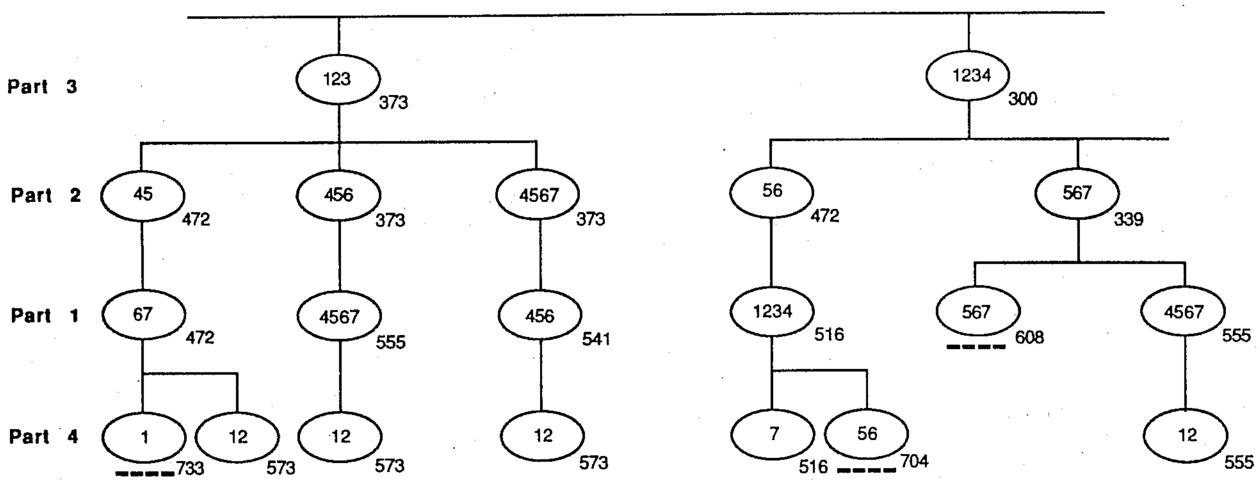
If all choices are carried through, considering time only, there are 12 acceptable assignments. It would now be necessary to calculate the time for each of these assignments and choose the smallest.

Figure 4

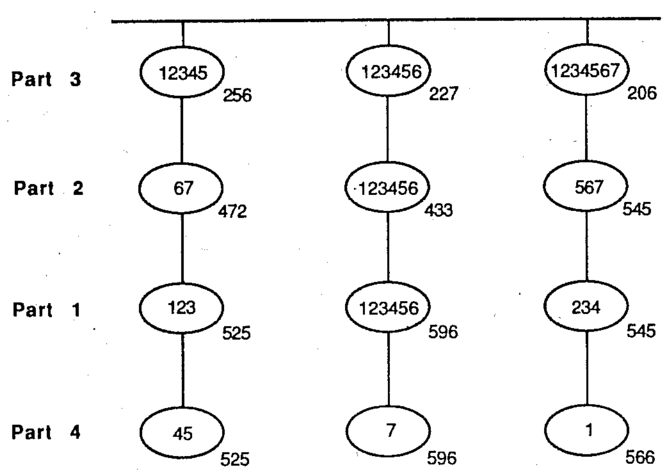


START
Part 3
1
960
12
520
123
373
1234
300
12345
256
123456
227
1234567
206
1234
5678
190
Part 2
34
520
345
520
3456
520
34567
520
345678
520
Part 1
56
520
567
520
67
520
345
541
3456
520
3456
520
345
520
Part 4
7
520
67
720
7
541
7
520
67
720
7
624
67
520

Figure 4



Part 3
123
373
1234
300
Part 2
45
472
456
373
4567
373
56
472
567
339
Part 1
67
472
4567
555
456
541
1234
516
567
608
4567
555
Part 4
1
733
12
573
12
573
12
573
7
516
56
704
12
555



Part 3
12345
256
123456
227
1234567
206
Part 2
67
472
123456
433
567
545
Part 1
123
525
123456
596
234
545
Part 4
45
525
7
596
1
566

The 12 choices are as follows:

Part 3	Part 2	Part 1	Part 4	
12	34	56	7	Cost = $88,100
12	3456	345	7	Cost = $98,600
12	3456	3456	7	Cost = $101,700
12	34567	345	67	Cost = $106,000
123	45	67	12	Cost = $91,800
123	456	4567	12	Cost = $101,700
123	4567	456	12	Cost = $102,300
1234	56	1234	7	Cost = $99,100
1234	567	4567	12	Cost = $104,100
12345	67	123	45	Cost = $99,700
123456	123456	123456	7	Cost = $124,900
1234567	567	234	1	Cost = $105,700

Some of the work can be avoided by computing the cost for one suitable branch and then eliminating a branch when the time for that branch exceeds the time calculated.

One might assume, for example, that it would be a good idea to put three people on Part 3. We might calculate the cost for

(123)—(45)—(67)—(12) .

The time here is 573 hours, and the cost is $91,800. As soon as the cost of any branch exceeds this amount, it can be cut off.

The least expensive branch turns out to be

(12)—(34)—(56)—(7) ,

for which the cost is $88,100.

The use of the branch and bound method may seem forbidding to students attacking it alone. It is certainly not necessary in order to solve the problem. However, if done as a class project or in a group, it is rather fun. A blackboard and colored chalk might be used to mark the branches with the class in groups doing the time and cost calculations on different branches.

Some of the additional questions in Section VIII give the students a chance to try this method in easier situations.

VI. The Computer Program

The computer program provided on the diskette is entitled "Least Cost." The assumption is made that only 8 workers are available.

There are two options in this program: (1) if the user inputs an assignment of workers for the training program, the computer will output the time and cost of the assignment; (2) if the user inputs a number of workers to be used in Part 3 and Part 2, and a time limit on the training program, the computer will output the assignment of workers with least cost. This option only allows as many as two workers on Part 1 and Part 4. If the training program cannot be developed in that time, the computer prints out an appropriate message. The second option may take a long time to run.

VII. The Written Report

The ability to communicate clearly, both orally and in writing, is an important skill in any area. Report writing is a regular part of the job of a person working in industry. One of the skills gained from the use of the AIM problems should be an increased ability to read with understanding and to write clearly and accurately.

In this AIM problem the student is planning the development of a training program. This role includes writing a report stating the work requested, the assignment of personnel decided upon, and describing the reasons for the decisions.

The following suggestions may be helpful to you in making report writing a meaningful experience.

1. It is important that the student know from the beginning that a written report is expected and what that report should cover. For this AIM module, the report might deal with Problem A4.

2. Writing the report should begin with the Body. (See Report Format, Appendix B.) Have the student write the Introduction as soon as the student realizes what is to be calculated. The introduction would include only three or four sentences describing the problem. For example: AT&T National Product Training Center, Dublin is preparing a training program for CCC. We are asked to plan how to budget time and money to meet a given set of criteria.

3. After the students have studied their resource books and watched the first video, they can begin the Discussion. What assumptions are made? What notation is used?

4. Have the student hand in the Introduction and the first part of the Discussion at this time. In this way the report is

actually begun, and you can be sure the student understands what is given and what is to be done.

5. As the student works on the problem, things needed in the Discussion can be listed. In this case the time chart (including totals only if desired) would be helpful.

6. Details of the decision process could be put in an Appendix. The best schedule of activities should be included under the heading Conclusions and Recommendations.

7. The opening material can now be added. The Glossary can be fairly short in this case. The Abstract should include a statement of the problem and the plan of procedure.

8. The References and Acknowledgements can be listed as the work proceeds so that they will be available at report time. Typists, teachers, or persons giving advice should be acknowledged.

Although students are rarely delighted at the prospect of writing a report, they are very proud when they see what they have accomplished! Compliment them if at all possible. Suggest that they keep the report for their portfolio for use when applying for scholarships or jobs.

VIII. Additional Questions

A. Budgeting Time

1. On Monday morning Jennifer is assigned an English paper due Friday. She has an algebra exam Friday morning and a track meet Friday afternoon. She needs to spend 2 hours in the library getting information, 4 hours writing her paper, and 1 hour typing it. She needs to study at least 4 hours for her algebra exam. Her track coach wants her to work out one hour on Monday, Tuesday, and Wednesday, and a half hour on Thursday. The total time she has free for study and workout each day is 4 hours.

    a. How should Jennifer budget her time Monday through Thursday?

    b. Is your answer the only possible one?

    c. Why do you think your answer is best?

    d. Is there time left over after the required work is planned? If so, what would you advise her to do with the extra time and why?

2. Two workers (W1 and W2) are assigned to Parts 1 and 2 of the development program discussed in the Student Resource Book. The charts below are two possible ways of assigning these workers.

PLAN 1	Part 1	Part 2
W1	*x*	*x*
W2	*x*	*x*

PLAN 2	Part 1	Part 2
W1	*x*	
W2		*x*

a. Calculate the real time required for each plan.

b. Which plan requires the least real time?

3. Repeat Problem 2 using Parts 1 and 3 in place of Parts 1 and 2.

4. Repeat Problem 2 using Parts 2 and 3.

5. Repeat Problem 2 using Parts 3 and 4.

6. Three workers (W1, W2, and W3) are to do Parts 1, 2, and 3 of the development program.

PLAN 1	Part 1	Part 2	Part 3
W1	*x*	*x*	*x*
W2	*x*	*x*	*x*
W3	*x*	*x*	*x*

PLAN 2	Part 1	Part 2	Part 3
W1	*x*		*x*
W2	*x*	*x*	
W3		*x*	*x*

PLAN 3	Part 1	Part 2	Part 3
W1	*x*		
W2		*x*	
W3			*x*

Which of the three plans requires the shortest time?

7. If n people are all assigned to work on each of the four parts, the formula for time required is

$$248 + (1/n)(2640) \text{ hours.}$$

If the workers are assigned in this way,

a. how many are needed to complete the work in 600 hours?

b. how many are needed to complete the work in 1/2 year?

B. Budgeting Cost

1. Tom makes $4 per hour working after school. He works 4 hours, 3 days a week, and 7 hours on Saturday. His transportation to

and from work costs $10 a week. His parents give him $1.50 per day to pay for his lunch at school. Tom prefers to eat with his friends at the Pizza Hut, which costs him $3 a meal. He is also hoping to buy a new bicycle in four weeks that will cost $200, and he is saving $15 a week for a fishing trip this summer. How should Tom set his budget for the next four weeks? The following chart might be used:

Distribution of Income:	Bicycle	Savings	Gasoline	Meals
Week 1	___	___	___	___
Week 2	___	___	___	___
Week 3	___	___	___	___
Week 4	___	___	___	___

Total Expenditures: ________

2. Two workers (W1 and W2) are assigned to Parts 1 and 2 according to the plans described in Problem A2.

   a. Calculate the cost for each plan.

   b. Which plan is less expensive?

3. In Problem A3,

   a. calculate the cost for each plan.

   b. Which plan is less expensive?

4. In Problem A4,

   a. calculate the cost for each plan.

   b. Which plan is less expensive?

5. In Problem A5,

   a. calculate the cost for each plan.

   b. Which plan is less expensive?

6. In Problem A6,

   a. calculate the cost for each plan.

   b. Which plan is less expensive?

   c. Can you guess which type of plan is less expensive for any number of workers?

7. a. Show that if two workers are assigned to the development of Parts 1, 2, 3, and 4, assigning all four to each part is the most expensive way.

b. Is this true for three workers?

c. Assume it is true that for n workers the most expensive way to assign workers is to assign all to one part. Show that if you add one more worker, so that you now have n + 1 workers, the most expensive way is to have all the workers work on each part. You might wish to use the formula

$$Cost = \$66,000 + n_1(3100) + n_2(3700) + n_3(2400) + n_4(2500) + n^*(1200).$$

NOTE: Parts a and c prove that for any number of workers assigned to the development of this training program, the most expensive way to do the work is to assign all the workers to each part. This method of proving a statement is called ***mathematical induction***.

C. The Branch and Bound Method

1. Assume that two workers are assigned to Parts 1 and 2. Use the branch and bound method to find the shortest time to complete these parts. Compare your answer with the answer found in Problem A2.

2. Assume that two workers are assigned to Parts 1 and 3. Use the branch and bound method to find the shortest time and compare with Problem A3.

3. In the discussion of the branch and bound method, Section V of the Teacher Resource Book, an example is given showing how two workers can be assigned to perform Parts 3, 2, and 1 in that order in the least time. Draw a diagram treating the parts in the order Part 1, Part 2, and Part 3. Is the final answer the same?

4. Is it possible for three workers to do all four parts in 900 hours? If so, how should they be assigned? Observe that it will be quite lengthy to carry all branches when three workers are used. However, a branch can be abandoned as soon as the time associated with that branch is more than 900 hours. If it is not possible, how many workers are needed and how can they be assigned?

D. Answers

A. *Budgeting Time*

1. a.

	Mon	Tues	Wed	Thurs	Total
English	2	2	2	1	7
Algebra	1	1	1	1	4
Track	1	1	1	1/2	3 1/2

b. The answer is not unique except for the track assignment. It is also important to do the library work on Monday and the typing on Thursday.

An argument for dividing the work the way suggested would be that study on Algebra is more effective when it is spread out rather than crammed on the last day. The ideas of the English paper can be worked out better with time to think about it between sessions.

The other extreme would be as follows:

	Mon	Tues	Wed	Thurs	Total
English	3	3	1	0	7
Algebra	0	0	1/2	3 1/2	4
Track	1	1	1	1/2	3 1/2

Students might argue that they work better when they concentrate on a single subject or when they work under pressure.

c. There are 1 1/2 hours available. Uses might be as follows: relax, more study for algebra, do other required work, recheck the English paper, etc.

2. a. Plan 1: 376 + 472 = 848 hrs
   Plan 2: max(872,696) = 872 hrs

   b. Plan 1

3. a. Plan 1: 896 hrs
   Plan 2: 960 hrs

   b. Plan 1

4. a. Plan 1: 992 hrs
   Plan 2: 960 hrs

   b. Plan 2

5. a. Plan 1: 720 hrs
   Plan 2: 960 hrs

   b. Plan 1

6 a. Plan 1: 269 + 339 + 373 = 981 hrs
   Plan 2: Max(376 + 520,376 + 472,472 + 520) = 992 hrs
   Plan 3: Max(696, 872, 960) = 960 hrs

   b. Plan 3

7. a. $248 + 2640/n = 600$

$$248n + 2640 = 600n$$
$$(600 - 248)n = 2640$$
$$352n = 2640$$
$$n = 7.5$$

Eight workers are needed.

b. $(1/2)\text{year} = 900 \text{ hours}$

$$652n = 2640$$
$$n = 4.049$$

Five workers are needed.

B. Budgeting Cost

1. Income:

Source	Amount
Work	$4 \times 3 \times 4 = \$48/\text{week}$
	$7 \times 4 = \$28/\text{week}$
Parents	$5 \times 1.50 = \$7.50/\text{week}$
	Total Income = $83.50/week
	Total Income, 4 weeks = $334

Distribution of Income:

	Bicycle	Savings	Gasoline	Meals
Week 1		$15	$10	$8.50
Week 2		15	10	8.50
Week 3		15	10	8.50
Week 4	$200	15	10	8.50

Total Expenditures: $334

2. a. Plan 1: $22200 + 27400 = $49600
   Plan 2: $19100 + 23700 = $42800

   b. Plan 2

3. a. Plan 1: Cost = $49000
   Plan 2: Cost = $43500

   b. Plan 2

4. a. Plan 1: Cost = $54200
   Plan 2: Cost = $48100

   b. Plan 2

5. a. Plan 1: Cost = $39800
   Plan 2: Cost = $24400 + 10500 + 1200 = $36100

   b. Plan 2

6. a. Plan 1: Cost = \$25300 + 31100 + 29200
= \$85600
Plan 2: Cost = \$22200 + 27400 + 26800
= \$76400
Plan 3: Cost = \$19100 + 23700 + 24400
= \$67200

b. Plan 3 is least expensive.

c. A plan as close to Plan 3 as possible is least expensive.

7. a. We can see this best by considering the formula for cost. Part of the cost is fixed, \$66000. The variable part of cost, VC, depends on the number of workers on each part.

$$VC = n_1(3100) + n_2(3700) + n_3(2400) + n_4(2500) + n^*(1200).$$

If the two workers are assigned to all four parts, we have $n_1 = n_2 = n_3 = n_4 = 2$, and $n^* = 0$. If the two workers are not assigned to all four parts, at least one of the numbers n_1, n_2, n_3, $n_4 = 1$ or 0. The number n^* may not be zero if the persons working on Part 4 are not also working on Part 3. An increase of 1 in n^* must be accompanied by a decrease of 1 in n_3. Thus the cost will decrease. Thus assigning two workers to each part is the most expensive assignment.

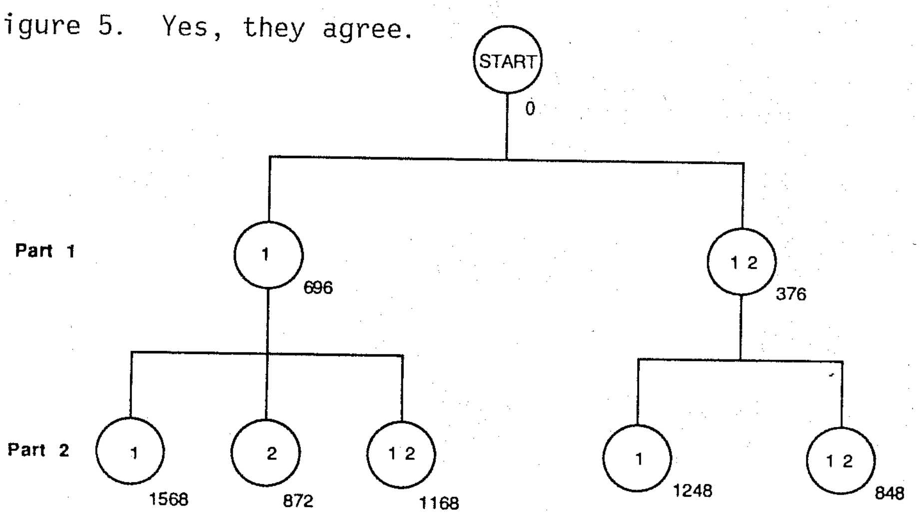
b. Yes, it is true for three workers.

c. If one worker is added to one part, the cost is increased by at least \$2400. The amount of increase in cost increases with each additional part that worker is assigned to. The most expensive assignment is to assign the worker to all parts. One might consider assigning the additional worker to Part 4 but not to Part 3. This would increase the cost by \$1200 and at the same time decrease it by \$2400. Thus to assign the worker to all four parts is most expensive.

C. *The Branch and Bound Method*

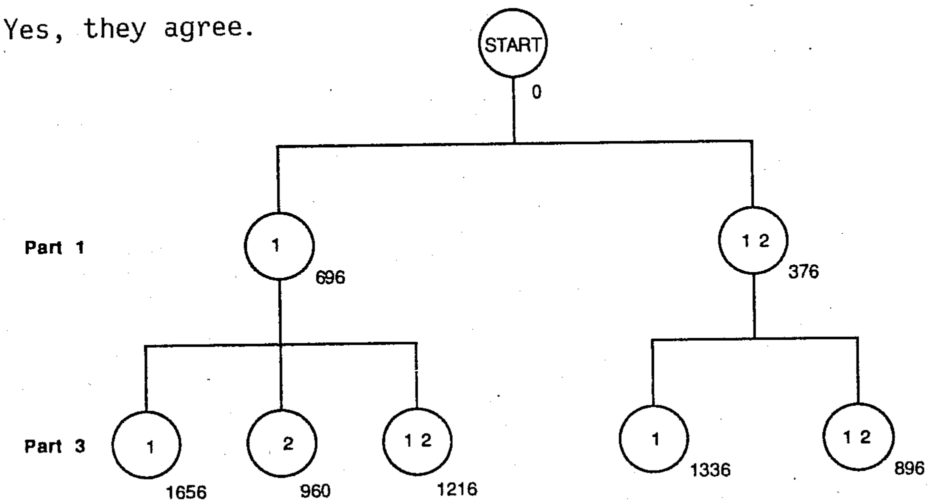
1. See Figure 5. Yes, they agree.

Figure 5



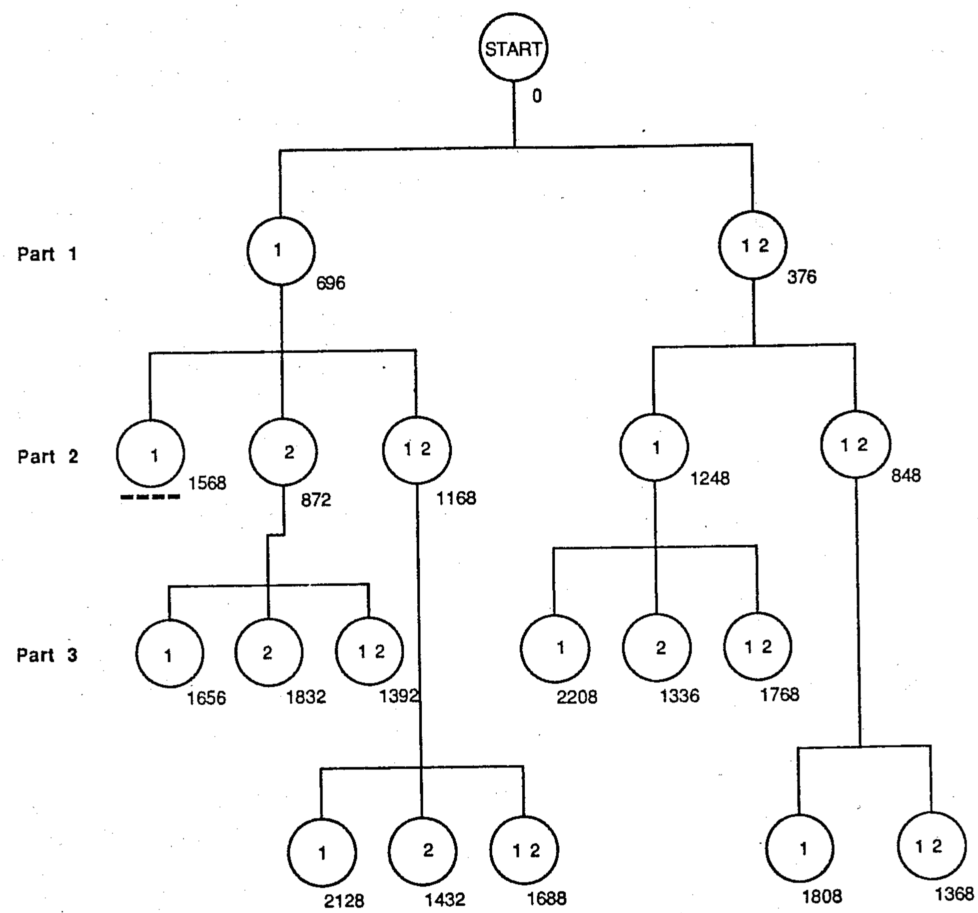
2. See Figure 6. Yes, they agree.

Figure 6



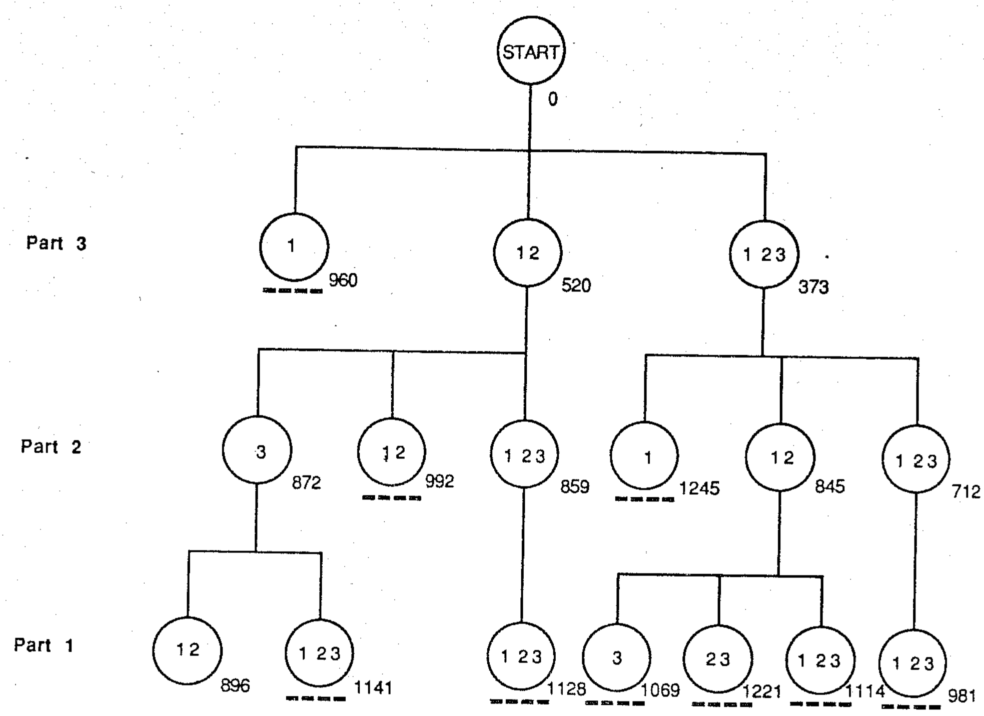
3. See Figure 7. Yes.

Figure 7



4. See Figure 8. No, four are needed. Use the branch

Figure 8



IX. Appendix

A. Glossary

bound - an upper limit for production time or for production cost.

branch - a path through the assignment chart indicating a possible assignment of workers.

CCC - Creative Communications Company, an imaginary company that produces a product using AT&T designed equipment.

optimization - the task of making something as effective as possible, in this case developing the training course in the least time or for the least cost.

part - a set of production activities assigned to a particular group of workers and making up one unit in the development of the training program.

real time - the time as measured by the clock and calendar.

W1, W2, ... - the letter W followed by a number is a designation of a particular worker assigned to the development program.

work hour - one work hour is produced by one person working one real-time hour.

work year - 1800 real-time hours.

B. Report Format

OPENING MATERIAL	
Title Page	
Table of Contents	Section headings and page numbers
Glossary	This should include only terms special to this problem.
List of Symbols	
Abstract	A brief statement of what the student was asked to do and the answer. (No more than four or five lines.)
BODY	
Introduction	Statement of what was assigned and why it was needed.
Discussion	The simplifying assumptions. An outline of the formulas given (and any formulas that were derived) and an outline of the procedure.
Results	A more detailed statement of the results than that given in the abstract.
Conclusions and Recommendations	If any value judgment can be given, it should be included here.
APPENDIX	
References	Any books or articles consulted.
Acknowledgements	Any people who helped.
Computation	A flow chart or program if needed. Detailed results or calculations.