M3 Challenge Champions (Summa Cum Laude Team Prize) - $20,000
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Team 481

Colorado River Water:

Good to the Last Acre-Foot

Summary

The arid region of the southwestern United States holds one of the most important bodies of water in the nation: the Colorado River, which provides the water to nearly 30 million people daily. The Colorado River Basin has been divided into Upper and Lower Basin regions since the signing of an interstate compact in 1922. One of the principal means by which water becomes available to the Lower Basin is by Lake Powell, a reservoir formed by the Glen Canyon Dam, which also works to furnish the necessary long-term storage needed to help the Upper Basin States use their apportioned share of the Colorado River water.

With our first model, we developed a simplified geometric model of the shape of Lake Powell in order to more easily simulate the effects of the drought on the volume of the water in the reservoir. We conclude that in the worst-case scenario, if inflow equals 39 percent of the average, then the lake would run dry in 3.5 years. If inflow equals the probable value of 83 percent of the average then the lake would almost reach capacity, and the high value of 137 percent of the average would yield maximum capacity.

From the second model, we conclude that the Glen Canyon Dam runs a lot better if the reservoir is full and that there is a large difference in the power generated between the three provided scenarios. This is due in part to the height of the reservoir as a direct result of the inflow and also to the fact that if the reservoir runs dry, the power plant would have too little output; conversely, a full reservoir would result in excessive power output.

In our third model, we attempt to analyze the agricultural data related to the economy of the states that make up the basin. We considered how much water was allocated to each state as a result of the 1922 Compact and how the agricultural GDP correlated to the amount of water allocated to agriculture.

We finally make recommendations on potential reductions to the amount of water that might be removed from the Colorado River in order to maintain the minimum capacity in Lake Powell.

Table of Contents

Table of Figures

I. Introduction

1. Background of the Situation

The Colorado River is one of the most important bodies of water in the United States today. More than 29.3 million people and 4.5 million acres of land in the United States and Mexico combined rely on the river on a daily basis, as the river provides 10 billion liters of water to the average resident of the basin per day for direct use, including activities such as drinking, cooking, flushing toilets, laundry, and the like [10, 24].

A complex group of laws and regulations referred to as "The Law of the River" govern the distribution of the water. The keystone of this is the 1922 Colorado River Compact, which divided the region at Lee Ferry between the Upper Basin (Wyoming, Colorado, Utah, and New Mexico) and the Lower Basin (California, Arizona, and Nevada). The compact stipulated that each basin would receive 7.5 million acre-feet per year (actually, to compensate for years with lower flow, it required that the Upper Basin ensure that the flow below Lee Ferry amount to at least 75,000,000 acre-feet over a rolling 10-year period.) The compact identified three main uses for water: domestic use (including household, stock, municipal, mining, milling, and industrial applications), hydroelectric use, and agricultural use [8].
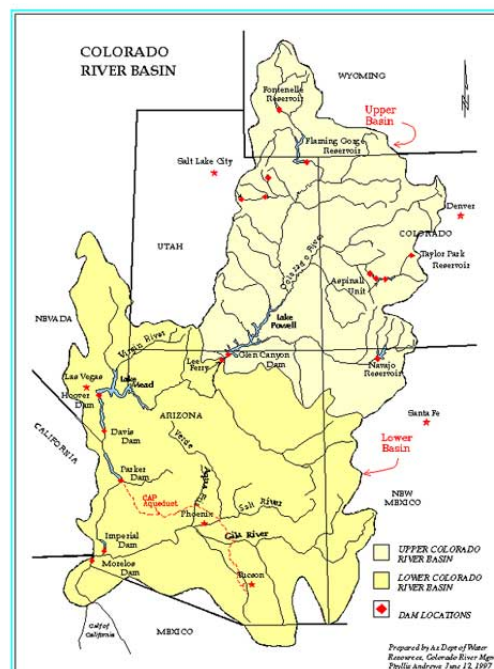

Figure 1. Map of the Colorado River and the surrounding basin and Compact states, delineating the Upper and Lower Basins.

The Compact also made provisions for sharing river water with Mexico, stipulating that if the United States chose to furnish Mexico with water, that water would come from surplus

water or, if no such surplus existed, would be drawn equally from the Upper and Lower Basins. (In 1944, the U.S. ratified a treaty with Mexico that allocated 1.5 million acre-feet, so each basin currently contributes .75 million acre-feet to Mexico.) [8].

The Boulder Canyon Project Act, which authorized the Hoover Dam and related irrigation facilities in the Lower Basin, specified the amount of water each Lower Basin state would receive: Arizona, 2.8 million acre-feet; California, 4.4 million acre-feet; and Nevada, 0.3 million acre-feet [8].

The Upper Colorado River Basin Compact of 1948 created the Upper Colorado River Commission and specified the percentage of the Upper Basin's 7.5 million acre-feet each Upper Basin state would receive: Colorado, 51.75 percent (3.88125 million acre-feet); New Mexico, 11.25 percent (0.84375 million acre-feet); Utah, 23 percent (1.725 million acre-feet); and Wyoming, 14 percent (1.05 million acre-feet). It also allocated 50,000 acre-feet to the portion of Arizona that lies within the Upper Basin [8].

Construction of the Glen Canyon Dam began in 1956, and on March 13, 1963, two diversion tunnels at the dam were closed to allow the lake to begin filling [15].

Lake Powell plays an important role in the Basin region. The lake serves as a reservoir so that, in case of a drought, the water the Upper Basin is obliged to send to the Lower Basin can come from Lake Powell as opposed to the Upper Basin states. This system has been working effectively; while drought has caused water shortages over the past five years, water users in the Upper Basin have not had to reduce water consumption because of Lake Powell's reserved water ("Drought in the Upper Colorado River Basin"). Lake Powell is not only crucial to reserving water but also necessary for generating hydroelectric power at Glen Canyon Dam and further down the river at Hoover Dam and for providing water imperative for the Lower Basin's economy.

Approximately 80 percent of what is left over after 2 to 3 percent of the water in the reservoir evaporates is employed for agricultural ends [1, 2]. In fact, irrigation coming from the Colorado River is what makes the normally arid lands surrounding the Lower Basin, whose waterways are fed by the water stored in Lake Powell, relatively fertile [11]. As a result, the agricultural productivity of this region is rather high, partly due to the warmer climate of the southwestern United States, providing the nation with an annual supply of fresh produce. Arizona, which is nearly entirely within the bounds of the basin, contributes $2.06 billion dollars of agriculture to the state's GDP. This livelihood is threatened by drought in the Colorado River Basin. Drought, therefore, has a direct effect on the economy of the Lower Basin, which comprises Arizona, Nevada, and California, the agricultural sector of which uses the water to grow crops that they sell to consumers and feed to livestock.

2. Restatement of the Problem

In modeling the effects of the current drought on Lake Powell, we must examine several significant issues. Our model of Lake Powell's percentage of capacity at the end of a five-

year period should account for high, low, and probable inflow values while taking into consideration small changes in the assumed inflow rates. We must consider the drought's implications for the economy of the Lower Basin, including hydroelectric power generation, and we must seek water-saving strategies to reduce outflow so that we can maintain the lake's minimum capacity.

II. Analysis of the Problem and the Model

1. Assumptions of the Model

The following factors will be assumed to be true throughout this analysis:

1. The efficiency of dam is assumed to be a constant, whose calculation is demonstrated below (see the calculations in the second model).
2. The average inflow rate of Lake Powell is 12.0 million acre-feet per year. A long-term drought has brought the lake's capacity to 60 percent, and the estimated future inflow ranges from an average 39 percent low to an average 137 percent high, with an 83 percent standard average.
3. For modeling purposes, the shape of the lake will be assumed to be a cone. (See "Design and Testing of the Model.")
4. The pressure of the water absorbed in the rocks below the Lake Powell reservoir is proportional to but less than the pressure of the water in the lake itself; this can be assumed because in areas where there is more pressure on the lake, more water would have seeped into those rocks, and likewise for areas with less pressure.
5. After being processed through a hydroelectric power plant, the entire volume of water utilized is included as part of the total outflow.

2. Addressing the Problem

In our first model we predict the volume of Lake Powell over the next five years based on the assumed inflow and expected outflow.

With our second model, we assess the effects of the current drought on hydroelectric power production at Glen Canyon Dam over a five-year range.

In our third model we consider the harmful effects of drought on the economy.
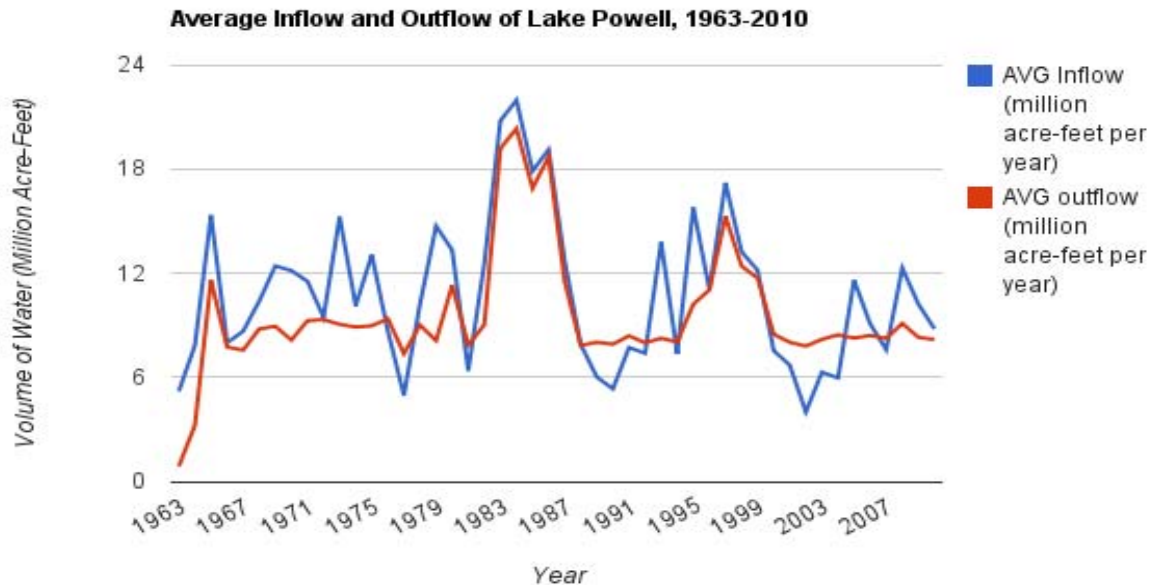
3. Design and Testing of the Models

Figure 2. Graph showing the patterns of the average inflow and outflow of Lake Powell over the period 1963–2010.

A. First Model: Addressing the Volume of Water in the Lake as a Function of Inflow

The change in volume in the reservoir is equal to the inflow minus the water leaving the reservoir from all sources: evaporation, seepage, and outflow. Thus, we let $V(t)$ be the volume of the reservoir, $I(t)$ be the amount entering the reservoir, $E(t)$ be the loss from evaporation, $S(t)$ be the loss from seepage, and $O(t)$ be the outflow through Glen Canyon Dam. Therefore, $\frac{dV}{dt} = I(t) - [E(t) + S(t) + O(t)]$.

Due to a series of agreements, at least 82.3 million acre-feet must be sent through the dam every ten years. We will assume that an average amount –– 8.23 million acre-feet, or 10,154.33 million cubic meters –– of water will be released annually; thus $O(t) = 1.015433 \times 10^{10}$.

The maximum depth of Lake Powell is 170 meters, and the mean depth is 40 meters [19]. Therefore, the depth ratio of a lake, defined as the mean height divided by the maximum height, is $\frac{4}{17} \approx 0.235$. The mean height multiplied by the base area of a solid equals the volume, so depth ratio $= \frac{\text{volume}}{\text{base area} \times \text{maximum height}}$. A hyperboloid surface would have a depth ratio between $\frac{1}{3}$ and $\frac{1}{2}$, a paraboloid has a depth ratio of $\frac{1}{2}$, and an ellipsoid has a depth ratio between $\frac{1}{2}$ and $\frac{2}{3}$ [9]. By contrast, a cone has a depth ratio of $\frac{\frac{1}{3}\pi r^2 h}{(\pi r^2)(h)} = \frac{1}{3}$. A sinusoid can have

a depth ratio as small as 0.297 [9]; however, this is only a small reduction and is not worth the complexity of sinusoidal models.

Lake Powell has a capacity of 2.6526×10^{10} cubic meters and a surface area of 65,843 hectares [19], or 6.5843×10^8 square meters, so it has an average height of $\bar{h} = \frac{2.6526 \times 10^{10}}{6.5843 \times 10^8} \approx 40.287$ meters. Because the maximum height is three times the average height, the maximum height of the reservoir is $3\bar{h} = 120.86$ meters. Note that this is significantly less that the true maximum value of 170 meters; this may explainable because there might be an unusually deep part of the reservoir that does not match our model.

In a cone, the radius of any cross-section is proportional to the distance of that cross-section from the vertex, so the area of a cross-section is proportional to the square of this distance, or $A = h^2 k_1$ for some constant k_1. Note that $k = \frac{A}{h^2} = \frac{6.5843 \times 10^8}{(120.86)^2} = 45075.73$, so $A = 45075.73 h^2$, or $h = \sqrt{\frac{A}{45075.73}} = \frac{\sqrt{A}}{212.310}$ regardless of the values of A and h.

Furthermore, $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}Ah = \frac{1}{3}\left(\frac{\sqrt{A}}{212.310}\right)A = \frac{A^{\frac{3}{2}}}{636.931}$. Rearranging gives us $A = (636.931V)^{\frac{2}{3}}$.

Because evaporation can only occur at the surface of a liquid, the rate of evaporation is proportional to the surface area of the lake. In water year 1997 (from 1 October 1997 to 30 September1998), 587,000 acre-feet, or 7.24251×10^8 cubic meters, of water were lost annually to evaporation [4]. In this year, the volume of Lake Powell was 21385702.44 acre-feet (2.63861×10^{10} cubic meters), so the surface area was 6.56112×10^8 square meters. Thus, the rate of evaporation per square foot of surface area is $\frac{7.24251 \times 10^8}{6.56112 \times 10^8} = 1.10385$ meters per year. We ignore seasonal variation in evaporation rate because we are concerned about long-term trends rather than short-term fluctuations.

The combined rates of seepage and evaporation have averaged 860,000 acre-feet (1.06108×10^9 cubic meters annually) [21]. Based on volume data [20] and our model, the average surface area of the lake over the past twenty years (1991-2010) has been 542,999,596.2 square meters. Thus, the average rate of evaporation has been $542,999,596.2 \times 1.10385 = 5.99391 \times 10^8$ cubic meters per year. It follows that, on average, the remaining $1.06108 \times 10^9 - 5.99391 \times 10^8 = 4.61692 \times 10^8$ cubic meters per year was lost due to seepage.

By Darcy's law, the rate of seepage is equal to the product of the area and the pressure difference between the lake and the rocks below [23]. Because we are assuming that the water pressure in the rocks is proportional to the water pressure in the lake, and water pressure is proportional to height ($P = \rho g h$), the rate at which water seeps out of the lake is proportional to the product of the height and the lateral area. Because the lateral area of a cone is proportional to its base area, this product is proportional to the volume of the

reservoir, so $S(t) = V(t)k_2$. The average volume over the twenty-year period 1991-2010 was 2.006507×10^{10}, so the seepage constant k_2 is equal to $\frac{4.61692 \times 10^8}{2.006507 \times 10^{10}} \approx 0.02300976$.

Finally, $I(t)$ is a constant, which we will call I. We are asked to test the model with I equal to 39%, 83%, and 137% of the historical average of 12.0 million acre-feet per year.

We can now write the differential equation for the water in the reservoir:

$$\frac{dV}{dt} = I(t) - [E(t) + S(t) + O(t)]$$

$$\frac{dV}{dt} = I - 1.10385A - 0.02300976V(t) - 1.015433 \times 10^{10}$$

Because $A = (636.931V)^{\frac{2}{3}}$, we have

$$\frac{dV}{dt} = I - 1.70336V^{\frac{2}{3}} - 0.02300976V - 1.015433 \times 10^{10}$$

We now examine each of the three cases using $V(0) = 1.62698 \times 10^{10}$ cubic meters of water, corresponding to the volume of water in the lake on 3 March 2011 [20]. The values of the low, most likely, and high estimates of inflow are 5.77427×10^9, 1.22888×10^{10}, and 2.028397×10^{10} cubic meters per year, respectively. We now solve the differential equation using an improved version of Euler's method, where we let $V_{n+1} = V_n + h\frac{f(t_n,V_n)+f(t_{n+1},V_n+hf(t_n+V_n))}{2}$ [25], where $f(t,V) = \frac{dV}{dt}$, and step size $h = 0.001$. Because the primary contributors to $\frac{dV}{dt}$ are I and -1.015433×10^{10}, both of which are constants, decreasing h would not significantly change the results.

In the low estimate, the reservoir is emptied within 3.560 years. In the most likely estimate, the reservoir's water increased to 2.45228×10^{10} cubic meters, about 92.45% of the lake's capacity. In the best-case scenario, the reservoir would reach its maximum capacity of 2.6526×10^{10} cubic meters in 1.066 years, less than a year and a month.

To determine the effect of a small increase in water inflow, we increase the most probable scenario's inflow to 84% of the historic average, or 1.24369×10^{10} cubic meters per year. In this scenario, the lake's volume increased to 2.52214×10^{10} cubic meters, or 95.08% of the lake's capacity. Thus, slight changes in water flow do not result in large increases in the lake's volume.

We tested the model by running it with known inflow and outflow models since 1963. Because the content, inflow, and outflow values are averages for the year, the data are not fully synchronized with each other because the inflow and outflow for a specific year affect the content of the reservoir for that year as well as the following year. The model tended to fluctuate more rapidly than the actual volume, but the trends for the model are similar to reality. However, some of the initial fluctuations may have resulted from the fact that the

sediments and rocks at the bottom of the reservoir absorbed a large amount of water before they were saturated. The model was effective for greater volumes of water, probably because the volume of the reservoir was capped at 2.6526×10^{10} cubic meters, but it often increased much more rapidly than reality for small volumes. The data for this testing is in Appendix A.

B. Second Model: Addressing the Effect of the Height of the Lake on Power Generation

An electrical generating plant converts the potential energy of water stored by the reservoir into electrical energy. If a certain mass drops a distance d, the potential energy lost by the mass is mgd (where $g = 9.8\text{m/sec}^2$). Because one cubic meter of water has a mass of 1000 kilograms, we can rewrite this expression for the total potential energy lost annually as $9800Od$ joules, where d the drop in height (in meters) and O is the outflow of water through the dam (in cubic meters per year). The Glen Canyon Dam has a hydraulic height of 579 feet [8], or 176.4792 meters, so the water falls 176.4792 meters if the reservoir is fully filled. This value is 55.619 meters greater than the height of the reservoir, so we can further expand this expression to $9800O(h + 55.619)$ joules. Because the maximum height equals $\frac{3V}{A} = \frac{3V}{(636.931V)^{\frac{2}{3}}} = \frac{\sqrt[3]{V}}{24.676}$, we can write the total loss in annual potential energy is $9800(O)\left(\frac{\sqrt[3]{V}}{24.676} + 55.619\right)$ joules. The electrical generating plant loses some energy to friction and seepage through the dam, so its efficiency is not 100%. We will assume that this efficiency is a constant e, so the total electrical energy generated is $9800(O)\left(\frac{\sqrt[3]{V}}{24.676} + 55.619\right)e$. However, electrical energy is usually measured in kilowatt-hours, and 1 kilowatt-hour equals 3.6 million joules (1 joule = 1 watt-second), so we divide this expression by 3.6 million to obtain $E = \frac{49}{18000}Oe\left(\frac{\sqrt[3]{V}}{24.676} + 55.619\right)$, where E is measured in kilowatt-hours per year.

In fiscal year 2007 (which, like the water year, lasts from October 2006 to September 2007), the power plant generated 3,454,846,789 kilowatt-hours of electricity [8]. In water year 2007, the reservoir volume was 14,776,353,822 cubic meters and the outflow was 11366.02 cubic feet per second, or 1.01499×10^{10} cubic meters per year. Thus, we have

$$3{,}454{,}846{,}789 = \frac{49}{18000}(1.01499 \times 10^{10})e\left(\frac{\sqrt[3]{14{,}776{,}353{,}822}}{24.676} + 55.619\right)$$

Solving for efficiency gives us $e = 80.637\%$. Based on our assumption that efficiency is constant, the plant is much more productive when the reservoir is full (as at the end of the high-inflow case) than when it is empty (as in case the low-inflow case). With an outflow of 1.01499×10^{10} cubic meters, the plant would generate 3.932×10^{10} kilowatt-hours if the reservoir were full but only 1.2392×10^9 kilowatt-hours with an empty reservoir (in which case the water would only fall 55.619 meters). In Utah, where the price of electricity is 8.162 cents per kilowatt-hour, this difference of 2.6928×10^9 kilowatt-hours translates to a loss of almost $220 million per year.

We can calculate the electricity generated by the plant by multiplying the outflow rate by the change in gravitational potential energy per cubic meter. We will assume that the outflow rate is 1.01499×10^{10} cubic meters per year unless the reservoir is full or empty, in which case the outflow rate will be equal to the inflow rate (if the reservoir is empty) or equal to the inflow rate minus loss to evaporation and seepage (if the reservoir is full).

For the worst-case scenario case, the outflow would be 1.01499×10^{10} for $t < 3.56$ (before the reservoir is empty) and 5.77427×10^9 for $t \geq 3.56$ (after the reservoir is empty). In the second case, $V = 0$, so $d = 55.619$. Thus, the average annual production of energy is
$\frac{1}{5}[\int_0^{3.56} \frac{49}{18000}(1.01499 \times 10^{10})(0.806371)\left(\frac{\sqrt[3]{V}}{24.676} + 55.619\right) dt + \int_{3.56}^{5} \frac{49}{18000}(5.77427 \times 10^9)(0.806371)(55.619)dt]$. This integral, computed by multiplying the flow rate by the reservoir's height (calculated from the volumes computed by the modified Euler's method every 0.001 year), amounts to 2,299,131,393 kilowatt-hours annually.

In the most probable scenario, the reservoir is never empty nor full, so the annual energy production is simply the outflow rate times the height:
$\frac{1}{5}\int_0^5 \frac{49}{18000}(1.01499 \times 10^{10})(0.806371)(\frac{\sqrt[3]{V}}{24.676} + 55.619)dt = 3,705,691,239$ kilowatt-hours.

In the best-case scenario, the reservoir fills after 1.066 years. When the reservoir is full, we have $V = 2.6526 \times 10^{10}$ and $\frac{dV}{dt} = 2.028397 \times 10^{10} - 1.70336V^{\frac{2}{3}} - 0.02300976V - O(t) = 0$, so $O(t) = 1.9658 \times 10^{10}$. Furthermore, $d = 176.4792$, the maximum possible drop. After this point, the dam generates $\frac{49}{18000}(1.9658 \times 10^{10})(0.806371)(176.4792) = 7.6154 \times 10^9$ kilowatt-hours annually. In this scenario, the dam produces 869,335 kilowatts of power, less than its maximum capacity of 1.32 million kilowatts. The total energy produced by the dam in this scenario is $\int_0^{1.066} \frac{49}{18000}(1.01499 \times 10^{10})(0.806371)(\frac{\sqrt[3]{V}}{24.676} + 55.619)dt + (5 - 1.066)(7.6154 \times 10^9) = 33,954,786,017$ kilowatt-hours, or 6,790,957,203 kilowatt-hours annually.

To determine the effect of a slight change in inflow on electric production, we reexamine the scenario in which inflow is 84% of the historic average –– slightly better than the midline prediction. In this case, the average annual energy production is
$\frac{1}{5}\int_0^5 \frac{49}{18000}(1.01499 \times 10^{10})(0.806371)(\frac{\sqrt[3]{V}}{24.676} + 55.619)dt = 3,719,386,880$ kilowatt-hours. Thus, a small change in inflow produces an even smaller change in the dam's power generation (the two values differ by only 0.3696%) because the excess water is stored in the dam's reservoir.

C. Third Model: Addressing the Effect of Colorado River Water on the Regional Agricultural Economy

	GDP (millions)	Water Allocation (Acre-Feet)	Water Allocation (m^3)	Percent GDP related to Agriculture	Agriculture GDP	Percent of Agricultural water from river	Annual Agricultural Value of the river	Percent of total GDP from the River
Arizona	$256,364,000,000.00	2850000	3515423244	0.80432510024808	$2062000000	80	$1649600000	0.64346008019847
Nevada	$126,503,000,000.00	300000	370044552	0.18418535528802	$233000000	83	$193390000	0.15287384488905
Colorado	$252,657,000,000.00	3881250	4787451391.5	0.72153156255318	$1823000000	90	$1640700000	0.64937840629787
Wyoming	$37,544,000,000.00	1050000	1295155932	1.13466865544428	$426000000	45.65	$194469000	0.51797624121031
New Mexico	$74,801,000,000.00	843750	1040750302.5	1.50131682731514	$1123000000	77	$864710000	1.15601395703266
California	$1,891,363,000,000.00	4402500	5430403800.6	1.22303333627654	$23132000000	12.95	$2995594000	0.15838281704781
Utah	$112,941,000,000.00	1725000	2127756174	0.48786534562294	$551000000	81.1	$446861000	0.3956587953002
Total	$2,752,173,000,000.00	15052500	18566.9853966	1.06643005363398	$29350000000	67.1	$19693850000	0.7155745659884

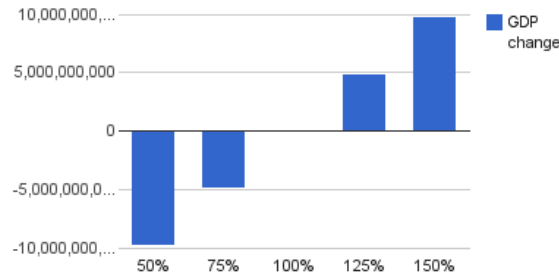*Figure 3. Data tables calculated and compiled to develop the third model.*

Figure 4. This graph shows the effect of water outflow availability from Lake Powell on regional GDP.

We used multiple unit conversions to estimate the effect of the Colorado River water used for agriculture on the GDP of the states in the basin. Initially, we determine the total GDP and GDP due to agriculture [26-29]. From this point, we divide water allocated from the river by total water usage to calculate the ratio of water used from the river to the total water usage by the states. Here, it can be seen that this percentage can be applied to the agricultural GDP for each state. Thus, we show that each state has a quantifiable amount of its GDP that is directly related to its usage of Colorado River water for irrigation and other agricultural purposes. This relationship is identified in the above chart. If the available water is reduced severely, there is a multibillion dollar loss of GDP. We calculate the change in GDP per year for 5 cases of water availability (see graph).

It thus needs to be ensured that there is enough reserve water so that in years when there are droughts, the reservoirs will be able to mediate these in order to prevent an economic shock to the region, especially considering the billions of dollars that go into the economy from the usage of water that passes through the Lake Powell reservoir.

In addition, we consider the microeconomic principle of derived demand, which holds that the demand for any given factor of production is derived from the demand for the good or service produced. Given that food has an inelastic demand by its very nature (represented

in Figure 2 by the nearly vertical line D), the demand for water is very high. Therefore, a reduction in supply (the leftward shift of the line S into S') as a result of unsuitable water leads to a higher price (P', compared to P), and due to the inelastic characteristic of the demand for food, the burden of any taxes that further elevate the price of food is undertaken nearly entirely by the consumers.
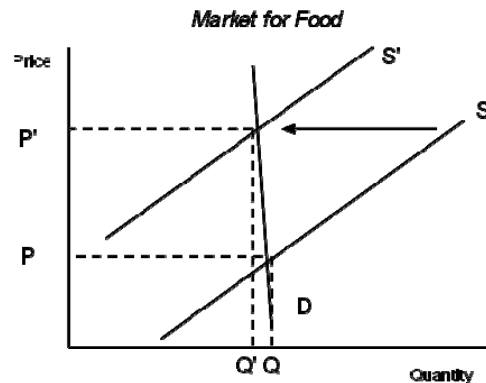

Figure 5. See the preceding description for an explanation of how this graph represents the impact of drought on the food supply originating in the Colorado River Basin.

4. Recommendations

We suggest that, due to the importance of a steady flow of water for the economy, the reservoir's content remains large enough that the outflow could be maintained at 8.23 million acre-feet (cubic meters) without the reservoir emptying even if drought limited inflow to 39% of the historic inflow. In our model of the worst-case scenario, the content of the reservoir two years before it dried up is   cubic meters. Therefore, we should attempt to maintain the reservoir at a level of at least   cubic meters, even if this requires some reductions in water quotas to either the Lower or the Upper Basin.

Since approximately 80 percent of the outflow from Lake Powell, after evaporation and seepage, is used for agricultural purposes, we recommend cutting down on water consumption in this sector to minimize the total amount of water removed from Lake Powell. To reduce the dependence of farmers on these waters, we offer the following suggestions:

- Minimize reliance on crops that require more water (including rice, produce grown in orchards, and Irish potatoes), and increase harvest of less water-dependent crops (e.g., soybeans, wheat for grain, and grain sorghum), and
- Use more efficient irrigation systems, including drip-trickle and low energy precision application.
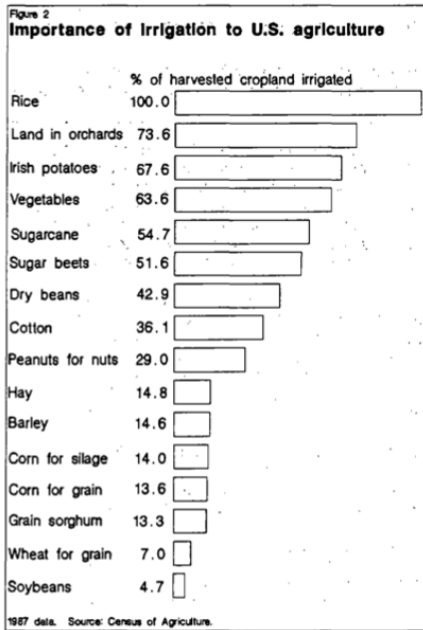
Figure 2
Importance of Irrigation to U.S. agriculture

% of harvested cropland irrigated

Crop	%
Rice	100.0
Land in orchards	73.6
Irish potatoes	67.6
Vegetables	63.6
Sugarcane	54.7
Sugar beets	51.6
Dry beans	42.9
Cotton	36.1
Peanuts for nuts	29.0
Hay	14.8
Barley	14.6
Corn for silage	14.0
Corn for grain	13.6
Grain sorghum	13.3
Wheat for grain	7.0
Soybeans	4.7

1987 data. Source: Census of Agriculture.

Table 2—Water use efficiencies of irrigation systems

System	Field efficiency
	Percent
Gravity:	
Flood	35-50
Furrow	55-70
Improved gravity[1]	75-85
Sprinklers:	
Big gun	50-65
Center pivot[2]	70-85
Side roll	65-80
Solid set	65-80
Hand move	55-65
Low Energy Precision Application (LEPA)	80-90
Drip-trickle	80-90

[1] Includes tailwater recovery, precision land leveling, and surge flow systems.
[2] Includes high- and low-pressure center pivot.

Of the total amount of water applied, 80-90 percent is used for plant growth. The rest runs off, evaporates, or percolates below the plant's roots.

Figures 6 and 7. These tables support the above recommendations.

III. Conclusion

The models we developed provide a fairly accurate picture of the situation in the Colorado River Basin. Our first model demonstrated the importance of the water inflow to the lake's volume. The second model took this idea forward by demonstrating how the water level not only ensures that there will be enough water necessary for consumption but also for hydroelectric power production. Our third and final model confirmed the economic importance of the Colorado River and its instrumental role in society today.

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Appendix A.

Comparison of the Model's Prediction of Volume Compared to Actual Volume

Year	Content (cubic meters)	Inflow (cubic meters per year)	Outflow (cubic meters per year)	Predicted content (cubic meters)
1963	570592623.4	6408389590	1072231914	570592623.4
1964	3293028462	9673311560	3990745031	5821986566
1965	6628218668	18917097938	14295133290	11292256012
1966	8353425215	9850718698	9548141316	15588586128
1967	7290084967	10676736909	9327688762	15522228069
1968	8336338104	12809892271	10832438224	16490222870
1969	10977857775	15310947929	11021922018	18055988595
1970	13396039926	14979733153	10042140830	21866927160
1971	16088633268	14217876639	11423140306	26229794981
1972	16097321973	11679840690	11498387158	26526000000
1973	18163990879	18795589993	11158838417	26086900369
1974	22254171596	12485350025	10965531541	26526000000
1975	23159724501	16101852729	11055490117	26526000000
1976	24016585620	10678505532	11565773465	26526000000
1977	20956854420	6113476239	9072640879	25032091160
1978	19339312199	12615772542	11111585824	21527819667
1979	23333051466	18138528835	10004865769	22509832574
1980	27236841731	16439391671	13939309648	26526000000
1981	25661179971	7864233958	9683199768	26526000000
1982	25902135531	15458726173	11124850493	24112524205
1983	28788889876	25632102372	23668386415	26526000000
1984	27812304606	27095280271	25069224833	26526000000
1985	27549741325	22060252923	20836098115	26526000000
1986	27779311116	23579848096	23062847555	26526000000
1987	27613383507	15644252896	14206934403	26418079921
1988	27202401156	9717562855	9656938403	26526000000
1989	24748470759	7414092710	9891736451	25967644055
1990	20660778729	6571683477	9748719196	22916980572
1991	18362897095	9507043171	10344566362	19246622437
1992	17491146910	9107897209	9848253346	17970259710
1993	20631925641	16997722600	10174537417	16819367951
1994	22288917721	9058018479	9910110474	23160090567
1995	24479113167	19454321517	12591431651	21778750263
1996	26058772017	13637473658	13609202494	26526000000
1997	26368571109	21219138893	18831766364	25935711901
1998	27136691281	16320581322	15300872140	26526000000

1999	27222646745	14999822204	14447877968	26526000000
2000	25878482399	9285742036	10449147338	26452568810
2001	23680963032	8258395620	9884027757	24687815572
2002	19471658129	4932572292	9606487996	22507668070
2003	15447122603	7760072806	10095110184	17369426264
2004	12265050258	7346116862	10417580104	14658593375
2005	13010429139	14292810856	10179655703	11283853521
2006	14602500267	11196283195	10374918784	15078210758
2007	14776353822	9360426145	10152626148	15535704196
2008	16095112227	15155371668	11226564157	14390087891
2009	17890952733	12600980426	10233759476	17930438482
2010	18142050778	10839021431	10084998665	19847522720
2011	17016073317	6162515320	14925888420	20128040095