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M3 Challenge Champions, $20,000 Team Prize

Car-Sharing is Caring

Executive Summary

We live in an era of unprecedented mobility where transportation is at a never-before-seen level. Vehicles are much more affordable than they were at their inception in the early 20th century, and public transport provides easy and economical means of travel for those who cannot afford or choose not to purchase a personal vehicle. The latest trend in the transportation industry is that of car-sharing. Realizing that purchasing and owning a personal vehicle are unnecessarily expensive, individuals are starting to turn to cheaper and more distributed means of paying for transport at a personal level.

In order to help demonstrate some of the different aspects of the car-sharing process, our team developed mathematical models that address some of the main concerns or factors influencing car-sharing companies' decisions. First, we developed a model that determines the proportion of drivers that fit into each category—low, medium, and high—for all combinations of hours driven per day and miles driven per day. We realized that much of the information regarding these two factors depended greatly on the amount of traffic in an area or city, which subsequently depended on the population density of that region. Hence, we created a function that gives the expected number of miles driven in a day based on the population density of the city or region and the number of hours driven in a day. We then created a means to produce a normal distribution around this expected average value and integrated a weighted cumulative density function of that distribution over time to get a table of proportions of drivers in each category. Next, we tested our model in two regions, New York City and Englewood Cliffs, a small suburban locale. Our model produced logical results in that the denser New York City had a larger proportion of cars moving a smaller distance and the less dense Englewood Cliffs had a larger proportion moving a larger distance in a given day.

We were also asked to create a model to rank four potential business plans for car-sharing companies in four different cities. We envisioned four scenarios that various consumers would fit into who could make use of a car-sharing service. We found an equation to model a "price" for the user that included both financial cost and opportunity cost, or a combination of a user's time spent and the value of an individual's time. We graphed the cost versus user salary for each scenario to determine which potential business plan would be most beneficial given a user's salary and scenario. This user-benefit model combined with the population density of a region gives the quantity of users for a car-sharing business in that region. Then we calculated the revenue and expenditure per user for each business model for a potential company. Combining this calculation with the number of users for a region gives the profit a business could expect. We applied this to the four cities and ranked them. This analysis would be highly beneficial to any car-sharing company wishing to expand to a new urban location.

Finally, we were asked to consider the effects of alternative energy vehicles and self-driving vehicles on the car-sharing market. We altered our model from Part II to adjust for the changes in usage, cost, and revenue to show the effects of these future changes. Any company wishing to develop a car-sharing business should consider these insights and future changes in order to keep their service relevant in the face-paced world of automobile technology.

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Restatement of the Problem

The problem asks us to do the following:
- Develop and test a mathematical model that accurately calculates the percentage of current American drivers in each category—low, medium, and high—for all combinations of miles driven per day and hours driven per day.
- Predict the expected participation in each type of business in four different cities.
- Predict the expected participation in each type of business in the four cities, taking into account the eminent emergence of innovative technologies and energy systems.

Part I: Who's Riding?

The two factors that have the greatest quantifiable impact on a car-sharing business's decision-making are the time the car is driven each day and the number of miles traveled per day. To develop our prediction for the proportion of current drivers in each combination of the possible categories of those two factors, we made the following assumptions:

A. Restatement of the Problem

In order to truly understand customer profile and demographics, one needs to be aware of two important factors—the average number of hours a driver drives per day and the average distance he or she travels in a day. This critical problem for car-sharing companies becomes more intricate when one realizes that those factors vary based on aspects of the setting, especially population density, as it gives a good idea of the amount of traffic in the area at any given time.

B. Assumptions

1) With regard to miles driven per day, we define "low," "medium," and "high" to constitute 0-20 miles/day, 20-40 miles/day, and 40+ miles/day, respectively. Using a histogram of 2009 data from the National Household Travel Survey (NHTS), we derived that 20 miles/day and 40 miles/day were roughly around the 33rd percentile and 67th percentile, respectively.
2) When one is "using" the car, one is actually driving the car.
3) The average driving speed of a car in the U.S. is 30.99 mph. This value was derived from NHTS data regarding the average miles per trip and average time per trip.
4) Traffic in a certain city, region, or locale can be considered as standardized over time in a day. This is a valid assumption because although traffic does follow a general trend in many locations, it is often unpredictable and always varies in areas on a regular basis due to spontaneous incidents such as car accidents, weather, and other extenuating factors.
5) Cities are uniformly distributed in terms of population. This assumption allows us to ignore differences in traffic and other conditions throughout the city, which makes application of the model possible on a large scale.
6) Across the US, the distribution of the amount of time individuals spent on the road is uniform. Though people may live in different regions with different population densities, they will work to minimize travel time. People living in communities with low population

densities will tend to drive further distances but will spend the same amount of time in transit as those in denser areas.

C. Developing the Model

The percentages we obtain depend extensively on our assignments of values to the arbitrary characterizations provided by the problem. More specifically, we need to define what "low," "medium," and "high" mean in the context of miles driven per day and hours traveled per day. Through one of our assumptions, we define "low," "medium," and "high" to mean 0-20 miles/day, 20-40 miles/day, and 40+ miles/day, respectively. For the hours/day factor, we took the data set from the 2009 NHTS study (sample size of 235333 individuals) and found the 1st and 3rd quartiles for the average amount of time individuals drove per day (NHTS). These values were 34 minutes and 111 minutes, respectively.
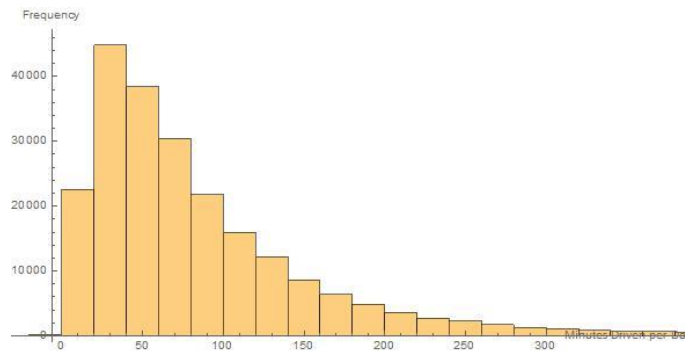

Figure 1.1 – Histogram displaying frequency of individuals (y-axis) who drive a certain number of minutes per day (x-axis).

Since we are trying to find the proportion of individuals who fit each of the 9 categories, we need to determine a relationship between M, the number of miles one drives per day, and H, the number of hours one spends in the car per day. Dimensional analysis yields the formula

$$M = Hv,$$

where v refers to the average velocity of the driver's car. This makes sense: the equation above essentially follows the rate equals distance times time equation. Hence, most of the difficulty comes with understanding how to model the average velocity of the car, as one has to take into account a variety of factors to develop an accurate prediction.

Above all other factors, the most prominent influence on velocity during the trip is the amount of traffic present. Since we assume that traffic in a location is standardized over time, we can view traffic as dependent upon the general amount of activity in an area, which can be simplified to population density. Ideally, when there is no activity taking place (i.e., population density = 0), one travels right at or slightly above the average driving speed, and as the amount of activity increases (i.e., population density increases), one's average travel speed reaches an asymptotically bounded minimum. Given our assumption that the average driving speed of a car is 30.99 mph, we can define v as

$$v = (1\text{-}T)V_0,$$

where V_0 refers to the average driving speed, 30.99 mph. T is the traffic factor and is equal to

$$T(p) = K - e^{-rp},$$

where p is equal to the population distribution of the area and r and K are positive constants less than 1. Therefore,

$$M = H(1\text{-}K+e^{-rp})V_0.$$

If the population distribution is 0, then $(1\text{-}T)V_0$ is greater than V_0. This is a valid result, since in an essential vacuum of people (e.g., a very sparsely populated and rural environment), drivers will go slightly faster than the average speed limit (which takes into account increased stopping times and greater periods of driving slow due to immediate traffic and lane switching, among other aspects of driving on a road occupied by other cars). Meanwhile, if the population distribution is very high, e^{-rp} is approximately 0, and the $(1\text{-}T)V_0$ approaches a minimum speed, which makes sense considering that the traffic would be so extensive that the car would not make it past the traffic (assuming the trip is of normal length).

Moreover, because drivers in all cities/regions will not all move the same distance in the same time period, we must see a distribution of values of distances traveled M over the time H. Because the number of cars moving in a given city/region will be large, and because some drivers will go farther than the expected value while others proceed less (potentially as a result of random occurrences like stop signs, traffic lights, accidents, etc.), which implies symmetry, we have the necessary conditions to use a normal distribution to represent the distances drivers travel over the time H.

In order to complete our function, we need to find the values of the constants K and r. We can do this via curve fitting with data relating to average driving speed, V_C, in various American cities (Infinite Monkeys). Using the population densities of these cities and the fact that $T = \frac{V_0 - V_C}{V_0}$, we can approximate a good curve that generally captures a wide range of cities' conditions.

City	Population Density (ppl/mi^2)	V_C (mph)
Albuquerque, NM	2908	31.2
Dallas, TX	3518	26.1
Washington, D.C.	9856	19.3
Seattle, Washington	7251	24.0
San Diego, California	4003	31.0

Table 1.1 – Sample of the city average driving speed vs. population density data.

Using curve-fitting techniques, we can derive a function of the form $K\text{-}e^{-rp}$, with K equal to 0.7 and r equal to 0.00009.
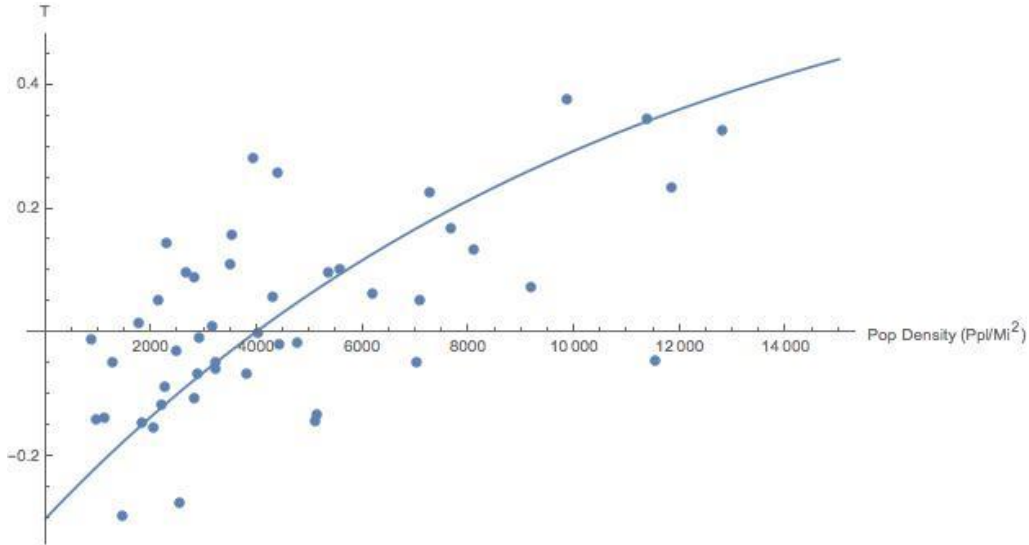
Figure 1.2 – Plot of the curve fit for *T(p)* along with the raw data for *T* vs. population density.

Accordingly,
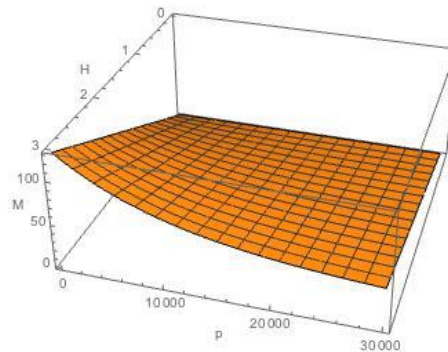
$$M(H, p) = 30.9*H*(0.3 + e^{-0.00009p}).$$

Figure 1.3 – 3D plot of miles driven per day as a function of population density and hours driven per day.

Our function for *M* provides us with an expected outcome for the number of miles one would travel in a particular region in a given time period.

In order to model a distribution, we need a standard deviation. The nationwide standard deviation for miles traveled per hour is 5.2645, and the average driving speed is 30.99 mph (NHTS). We can scale this standard deviation value up or down to the level of a specific region's traffic based on the proportion of the city's average speed to the national average driving speed. We obtained the city's average speed through our function for M by inputting 60 minutes for *H* in the function for *M*, we can determine the number of miles one could travel (on average) in that region. Based on this knowledge, we can make our normal distribution in the following manner:

$$G(x, t) = e^{-\frac{(x-\frac{t}{60}b)^2}{2(\frac{t}{60}c)^2}},$$

where *b* is equal to the expected average speed returned by our function for *M* and *c* is equal to the average standard deviation nationwide scaled to the specific region. Therefore, *(t/60)*b* refers to the expected number of miles traveled in a given number of minutes, while *(t/60)*c* refers to the standard deviation of a distribution of those miles traveled. Based on this, the proportion of vehicles traveling more than *y* miles in a given time period is given by

$$\int_{t_{initial}}^{t_{final}} CDF[G(x,t),y] \, dt,$$

where CDF refers to the cumulative distribution function (the probability that *x* will have a value less than or equal to *y*). However, the number of people who travel for 35 minutes likely will not be the same as the number of people who travel for 3 hours. Therefore, we are going to need a weighing factor for periods of travel time that is based on their relative frequency. In order to determine the distribution of travel times, we looked at data from a NHTS 2009 survey, which listed the average daily travel times for a million survey participants (NHTS).
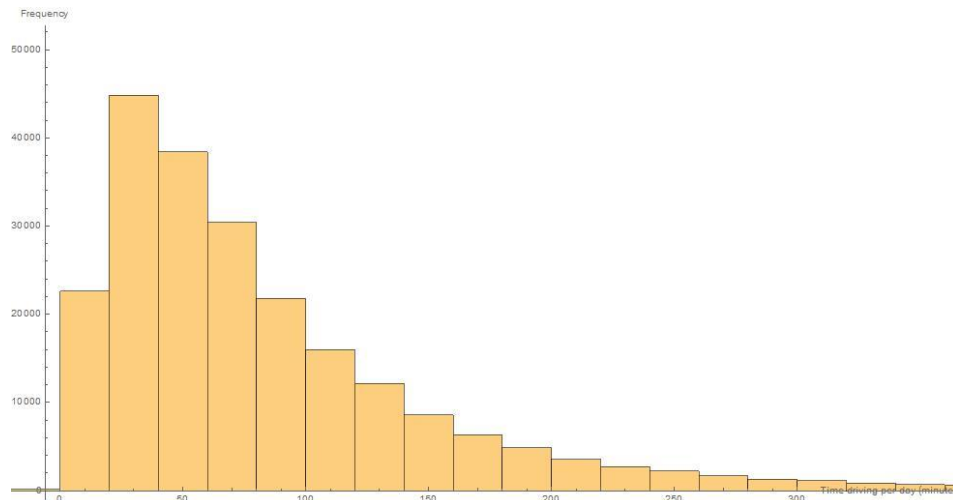
Figure 1.4 – Histogram of the NHTS travel time data.

The histogram above allows us to derive a cumulative density function that provides us with the proportion of a population (based on our assumption that all Americans share the same daily travel times) that spends under *x* hours driving per day. The CDF is given and plotted below:

$$W(x) = CDF[dist, x],$$

where the variable *dist* refers to the distribution of data shown in the histogram.
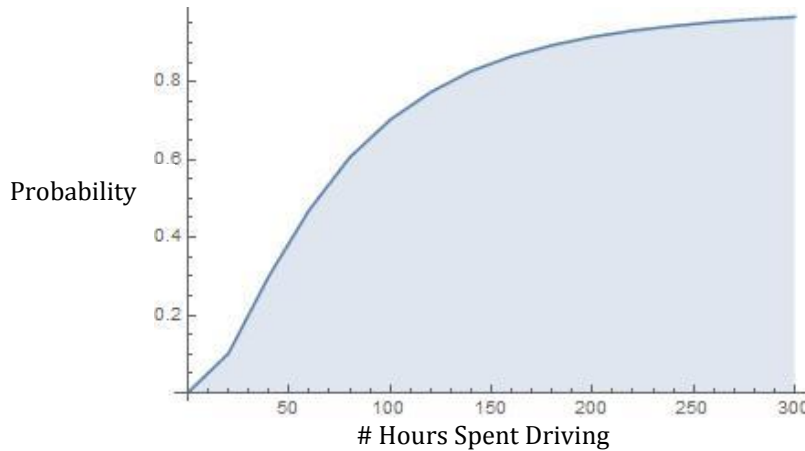
Figure 1.5 – Plot of the CDF function, giving the probability that a driver will spend a number of hours driving.

The CDF function can be used as our weighting factor because it gives us the relative frequencies of various travel times. As a result, we can use the following term in order to express the relative weight of a certain period of travel time from t_1 to $t_1 + 1$:

$$CDF[dist,\ t_1+1] - CDF[dist,t_1].$$

Thus, we can derive the final function that provides us with the percentage of current U.S. drivers in each category—low, medium, and high—for all combinations of the two specified factors, daily travel time and daily mileage:

$$\int_{t_{initial}}^{t_{final}} (CDF[dist, t + 1] - CDF[dist, t])\ CDF[G(x, t), y]\ dt.$$

The amount of travel time we wish to check relative proportions for is given by t, and the distance we want to see if our population crossed (e.g., 20 miles, 40 miles) is given by y.

D. Validation of the Model

While developing the model for $T(p)$, we calculated our root mean square error, which came out to be 0.167451. Given the large variance within the data points, however, this value is acceptable, as no model can effectively account for all of the variance in the data without resorting to a curve fit that cannot be extrapolated accurately. Hence, the fit function for $T(p)$ does a relatively good job at approximating the trends in the data.

In order to further validate the accuracy of the model, we conducted a sensitivity analysis. Because only two constants were solved for in the model (K and r), we decided to vary K and observe the changes in the value of $M(1, 2000)$, representing the number of miles per day given that the driver drives for 1 hour in a region with a population density of 2000. Table 1.2 displays the results.

K	% Difference from K = 0.7	M(1, 2000)	% Difference from M(1, 2000) with K = 0.7
0.8	14.29	31.9898	8.81
0.75	7.14	33.5348	4.40
0.7	0	35.0798	0
0.65	7.14	36.6248	4.40
0.6	14.29	38.1698	8.81

Table 1.2 – Sensitivity analysis with respect to *K*.

The sensitivity testing reveals that changes in *K* corresponds to about half as significant a change in the value of the model. This shows that the model is robust and not very sensitive to changes in *K*. We did not vary *r* because as the coefficient in an exponential, varying *r* would have required us to alter *K* in order to retain any accuracy. If given more time, we would have experimented with altering *r*.

E. Results of the Model

We used our model to determine the various proportions of a population in a very dense city: New York City, which has a population density of 26,403 people per square mile. Thus, we use the integral given at the end of section B to calculate the percentage of drivers in each category in New York City. Below is the table of output values.

	Low Driving Time (< 34 min/day)	Medium Driving Time (34-111 min/day)	High Driving Time (> 111 min/day)
Low Mileage (< 20 miles/day)	24.20%	35.56%	0.17%
Medium Mileage (20-40 miles/day)	0%	14.20%	11.51%
High Mileage (> 40 miles/day	0%	0.0098%	13.90%

Table 1.3 – Results for New York City.

Next, we used our model to determine the various proportions in a sparser suburb: Englewood Cliffs, New Jersey, a small town with a population density of 2506 people per square mile.

	Low Driving Time (< 34 min/day)	Medium Driving Time (34-111 min/day)	High Driving Time (> 111 min/day)
Low Mileage (< 20 miles/day)	21.51%	2.03%	7.79E-5 %
Medium Mileage (20-40 miles/day)	2.69%	24.86%	0.028%
High Mileage (> 40 miles/day	2.43E-8 %	22.88%	25.55%

Table 1.4 – Results for Englewood Cliffs.

The analysis of the results shows that the model makes sense. In a big city with a high population density, a large amount of the cars travel a low mileage, regardless of the amount of time they spend on the road. 59.93% of cars travel less than 20 miles, which is sensible given that heavy traffic limits much of the progress that cars can make. As a result, big cities tend to have a lot of cars traveling a low number of miles of day, while a very small percentage of cars travel more than 40 miles.

Meanwhile, in a suburban area such as Englewood Cliffs, more cars travel a high number of miles per day because of the reduced traffic in the region. Though some cars do travel a low amount of miles, the proportion of cars that travel more than 40 miles is certainly higher in Englewood Cliffs than in the Big Apple.

F. Strengths and Weaknesses

While our curve fit rather accurately describes the average speed of cars in New York City (our regression indicates a speed of 15.73 mph compared to a calculated average speed of 17.6) (Infinite Monkeys), our curve fit is not an extremely accurate model of average car speed vs. population density. With an RMSE of 0.1675, we have an average error with our T value of 0.1675. The reasons for this large error were addressed earlier, but it remains a fly in the ointment in our model.

However, our model takes into account population density as the major factor in traffic and car speed. In addition, our model is highly intuitive as more dense cities lead to lower traffic speeds. Moreover, our model is adaptable to many different urban or rural landscapes. We have gathered relevant and accurate data for both urban and suburban areas with our model. Furthermore, even though we did not calculate proportions for American drivers as a whole, our model can be applied to any individual setting in America and is therefore much more specific and powerful in realistic applications by car-sharing and other interested companies.

G. Summary

Simple dimensional analysis leads us to understand that the average miles driven is a product of average velocity and hours driven per day on average. Since each region's average velocity depends directly on its traffic, which is dependent upon its population density, we can state the expected miles driven as a function dependent on population density and hours driven in a day. In order to form a distribution around the expected value for miles (and therefore, the expected value for mph) that this function delivers, we can use the nationwide average standard deviation of velocity scaled down from the average nationwide driving speed to the driving speed in the particular city or region being examined to produce a normal distribution. Integrating a cumulative density function of this normal distribution, along with the weighting of the relative frequencies of different travel times through another cumulative density function, allows us to calculate the proportions of a population in each category—low, medium, and high—for all combinations of the two specified factors, hours driven per day and miles driven per day. The results of our model show that less dense regions and cities have a greater proportion of cars traveling higher numbers of miles per day, while denser cities and areas have a higher percentage of cars traveling a smaller number.

Part II: Zippity Do or Don't?

A. Restatement of Problem

A car-sharing transportation method can be defined as any form of transportation involving privately operated vehicles that does not involve a single owner of a vehicle. Car-sharing can help people avoid the costs of owning a vehicle, including insurance, maintenance, and gas. Alternately, car-sharing can be seen as a step up in luxury compared to public transportation; having the privacy of one's one car is seen as favorable to dealing with the chaos of a subway, train, or bus system. Today, roughly 10% of people choose a shared commuter experience over vehicle ownership (Automotive IQ).

As more people choose car-sharing options over POVs (privately operated vehicles), car companies lose vehicle sales. In America, one study estimates that 1.2 million vehicle sales could be lost by 2020 (Automotive IQ). In light of this, many car manufacturers, including Audi, Daimler, GM, and BMW have ventured into the business of car-sharing. As companies attempt to implement car-sharing programs, it is important to consider which car-sharing options will work best for various locations.

There are several viable car-sharing programs. The classic car rental model, popular for decades with travelers emerging from airports, involves a "round-trip" rental, in which the consumer rents a vehicle from a location for a certain amount of time and returns the vehicle to the same location. The pricing can be by hours, days, miles driven, or a combination of the three. Another, newer car-sharing program involves one-way trips. In the floating system, a user picks up a car and can park it anywhere with a certain zone. This eliminates the need for the user to return the car to the original location and does not require the user for pay for time spent not driving the car. The one-way station method is similar, but the user must drive the car to a specific station run by the same car-sharing company. Finally, in a more literal interpretation of "car-sharing" multiple people can jointly own a vehicle and share its costs while using it exactly like a POV.

B. Assumptions

1) The companies do not encounter implementation issues (e.g., implementation or construction costs or malfunctions). These are not within the problem parameters.
2) People are willing to walk/bike ½ mile to reach a car-sharing station. Even sedentary people walk 1000-3000 steps a day, and one mile is about 2000 steps. Therefore, most healthy people would have no problem walking up to ½ mile to reach a transportation station (The Walking Site).
3) Weather is not a factor. Weather varies throughout the year, and on a day-to-day basis is essentially random. Weather hazards will impede any sort of transportation and does not impact car-sharing transportation significantly more than other transportation methods.
4) Users do not drive one-way between cities. In order to analyze one-way car-sharing models for a particular city, the city itself should be isolated with respect to other car-sharing cities.
5) Users do not have car crashes, lose insurance documents or keys, park in unauthorized locations, drive without a license, or drive with impaired senses and functionality. In

these instances the cost for any given model skyrockets, and these data points become outliers.

6) Within the central area of a city, the population is uniformly distributed. We assume that the car-sharing company would put their stations in ideal locations, but between two cities the idealness of various locations would differ, on average, by a factor of the average population density for the major metropolitan area.

7) Across cities, bus fares are constant. We researched bus fares for each of the four cities we are investigating. The differences between cities are negligible in our calculations, so for the sake of simplicity, we will assume they are constant.

C. Design and Model

We decided that the profit a company makes from a car-sharing business in a given region is determined by

$$Profit = (revenue - cost) * usage,$$

where profit is profit, revenue, and cost are all per user, and usage is a function of both population density around a car-sharing station and the feasibility of the car-sharing method for an individual. Feasibility can be measured in terms of user price UP, which is a sum of opportunity cost and monetary cost. Opportunity cost is the amount of time T spent in commute, multiplied by individual salary S. Monetary cost M is the cost of a certain means of transportation.

$$UP = T * S + M.$$

We assume that the user wishes to save time and money and that time saved is more valuable if the user makes a high hourly wage, while money saved is more valuable if the user makes a low hourly wage.

We decided that a user could fall generally into one of four scenarios involving transportation.

Scenario I: A user needs a vehicle for only one continuous week a year but has the vehicle 24/7 during that week. For example, this might be a user who uses public transportation within his or her city but takes a week off a year to go on a road trip. Users in this general scenario would live in cities with very good public transportation. We will consider this scenario to be 4 one-way trips, for the sake of comparison. This would be a round-trip scenario.

Scenario II: A user only needs a vehicle for one continuous 3 hour period once a week. For example, this might be a user who uses public transportation for daily commute but needs a vehicle to carry groceries and other goods home once a week. Users in this general scenario would also have good public transportation in their cities. We will consider this scenario to be 4 one-way trips, for the sake of comparison. This could be either a round-trip scenario or a one-way scenario.

Scenario III: A user needs twelve 25 minute rides a week. This user would be someone who commutes to their day job every day and works for about 8 hours but also has a

weekend excursion of about 5 hours. We will consider the weekend excursion to be 2 one-way trips, for the sake of comparison. This would be either a round-trip or a one-way scenario.

Scenario IV: A user needs two 25 minute rides a week. The user would stay at their destination for about five hours. This would be someone who uses public transportation to get to work each day, but takes a weekend excursion. This would be either a round-trip or a one-way scenario.

To model cost of various car-sharing methods for each scenario, we looked at the pricing of existing car-sharing companies. For the round-trip program we looked at two different pricing models that differed significantly. The pricing models are described below:

Car-sharing method	Pricing method
Zipcar round-trip Monthly	$25 initial fee + $7 per month + $9.25 per hour or $84 per day
Hertz round-trip	$36 per day
One-way floating: Car2go	$35 + $0.41/min, $14.99 per hour, or $84.99 per day
One-way station: Zipcar Boston	$25 initial fee + $7 per month + $5 per 30 min
Joint ownership: 2 people	$4349 per year
Joint ownership: 3 people	$2900 per year

Table 2.1

	Scenario I ($/year)	Scenario II ($/year)	Scenario III ($/year)	Scenario IV ($/year)
Round-trip Zipcar	697	**1552**	26083	2995
Round-trip Hertz	**252**	1872	11232	1872
Car2go Floating	630	2373	6396	1066
Zipcar Boston station	1705	1669	3145	**545**
Joint ownership 2 people	4349	4349	4349	4349
Joint ownership 3 people	2900	2900	**2900**	2900

In the data above, the bolded values are the most cost-effective option for each scenario. Additionally, an average car commute time is 23 minutes, and an average bus commute time is 53 minutes. Our used commute time baseline was 25 minutes, so assuming proportionality, the bus commute time would be 58 minutes.

However, ride sharing is still less efficient than personally owning a vehicle. If an individual chooses to use a car rental service, he or she must travel (we will assume by foot) to the car rental station before he or she can begin driving. If an individual chooses to jointly own a car with others, the vehicle may not be available at any given time. Thus, we have determined additional times required for each method of car-sharing in minutes per one-way trip.

In determining these values, we assumed the following:
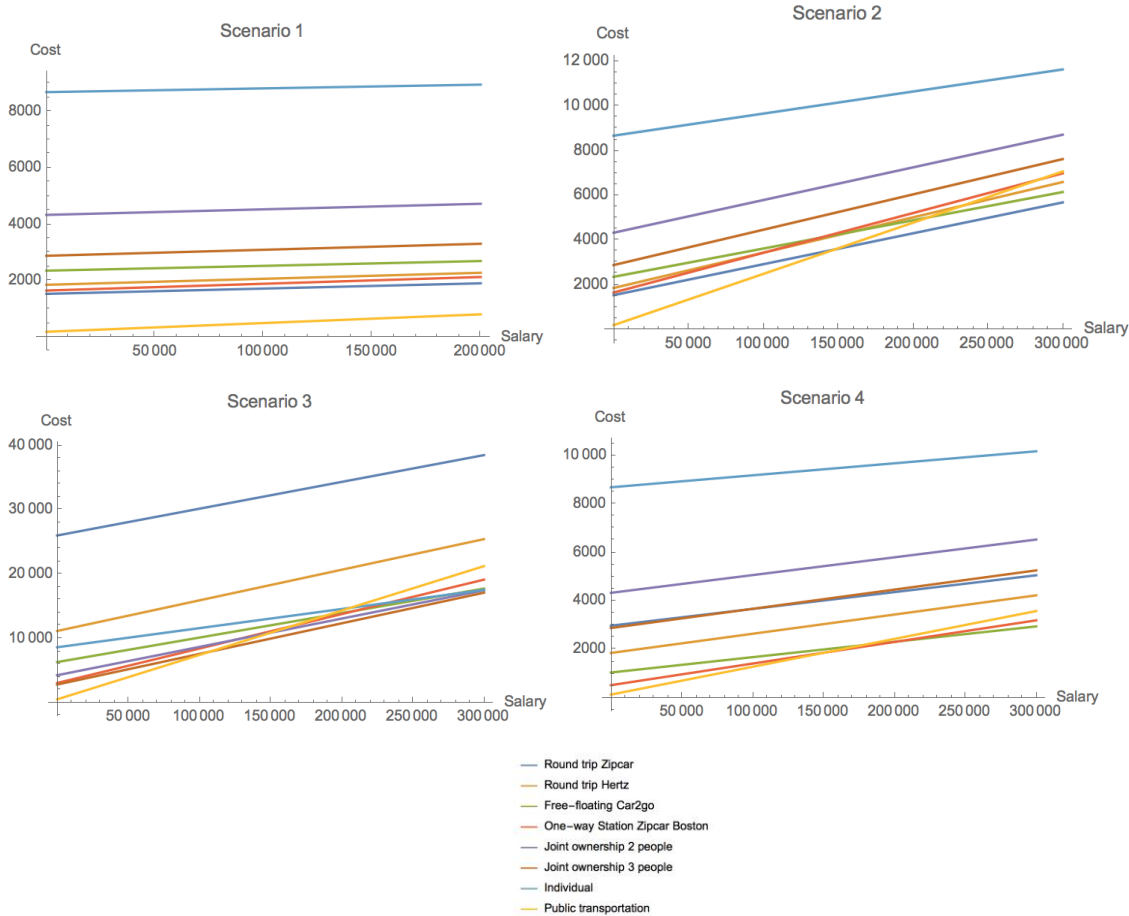- An individual can walk ½ mile in 10 minutes.

- A round-trip Zipcar station will be located at a distance of ½ mile at any given location. An individual has to walk ½ mile from his or her house to the rental station and 0 miles from the parking spot to the final destination.
- A round-trip Hertz station will be located further away. Hertz is less developed of a car-sharing service, so their stations are more spread out. An individual will need 15 minutes rather than 10 to walk to a Hertz station.
- One-way, free-floating rental sites are more abundant throughout the city. Thus an individual will walk a lesser distance (and for a smaller duration of time) from his or her home to a free-floating rental sites.
- One-way station model rental sites require ½ mile of walking from an individual's house to the site and ½ mile of walking from the parking site to the final destination.
- With joint ownership with a second person, there is a 1/10 chance the car will not be readily available for use.
- With a third person, there is a 1/8 chance the car will not be readily available for use.
- Each "trip" taken by your car-mate is assumed to be 2 hours long.

Car-sharing method	Additional time required per usage (minutes)
Round-trip Zipcar	10
Round-trip Hertz	15
Free-floating Car2go	7
One-way station Zipcar Boston	20
Joint ownership 2 people total	12
Joint ownership 3 people total	15

Using these assumptions, we calculated the time in minutes spent commuting per year:

Transportation method	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Round-trip Zipcar	924	6484	20592	3432
Round-trip Hertz	1064	7904	23712	3952
Free-floating Car2go	840	6240	18720	3120
One-way station Zipcar Boston	1204	8944	26832	4472
Joint ownership 2 people	980	7280	21840	3640
Joint ownership 3 people	1316	9776	29328	4888
Individually owned	644	4784	14352	2392
Public transportation	1484	11024	33072	5512

Now, using the equation $P = T * S + M$, we plugged in numbers for T and M for each scenario and transportation method. We let S be an independent variable and plotted the relationship between S and P for each transportation method in each scenario.

Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4

Legend:
— Round trip Zipcar
— Round trip Hertz
— Free–floating Car2go
— One-way Station Zipcar Boston
— Joint ownership 2 people
— Joint ownership 3 people
— Individual
— Public transportation

As the plots demonstrate, the lines do not intersect until the salary exceeds 50,000. Thus, the relative feasibility of each the options do not change until salary exceeds 50,000. Because the salary is multiplied by commute time, when salary is relatively low (under 50,000), the predominant factor in determining price P is the monetary cost M, as outlined in Table 2.1. As the median per capita salaries of the four cities range from $21,000 to $27,000, the relative feasibility of the car-sharing options for an average resident is the same as outlined in Table 2.1. However, at under 50,000, public transportation is still by far the cheapest option.

However, as the plots indicate, certain ride sharing options become much more economically advantageous when individual salaries increase. For Scenario 2, as individual salary increases, the most economically advantageous options are individual ownership and free-floating Car2go. For Scenarios 3 and 4, joint ownership and individual ownership, and free-floating Car2go and one-way station Zipcar Boston are most advantageous.

Now we apply need to calculate the (revenue – cost) per user for each case in order to figure out the company profit. For Zipcar, we found that they have a ratio of 90 users for every car (Zipcar Wikipedia). We assume that these cars are used 75% of the time and that gas costs $1.71 per gallon (Fuel Gauge Report). On average, we found that it takes $8698 a year to maintain a sedan, including fuel. Since Zipcars are used much more frequently than regular cars, however, we will assume that it still costs more to maintain these cars, so we estimate $10,000 per year (AAA Newsroom). From this we have the cost equation:

$$Expenses\ per\ member\ per\ year = \frac{(\$10,000 + \frac{\$1.71}{gallon}\frac{average\ miles\ driven\ per\ year}{average\ miles\ per\ gallon})}{90\ members}.$$

From Part I we found that the average miles driven per day are 28.97 miles, which translates to 21148 miles per year. The average miles per gallon of a US car is 25.5 mpg (Automotive News). Taking this all into account we come out with an expense of $126.9 per member per year. Each car-sharing option uses a different pricing model and will generate different revenue per user. Since the number of car-sharing users has doubled in the past five to six years, using an exponential growth rate we can deduce that the growth rate is 10%. This means that in any given year, about 12% of users are new.

For the Zipcar round-trip model, the average revenue made per user per year is given by

$$\$25 * \frac{0.12}{user} + \$7 * \frac{12}{user} + \frac{0.75*24\frac{hours}{day}*365\frac{days}{yr}*\frac{\$9.25}{hour/car}}{90\ users/car} = \$762.$$

Hertz rents cars by day more often, so we can estimate that their cars are rented out more often since they can be rented out even when the user is not driving them. Therefore, we can say that 85% of Hertz cars are rented out at any given time. Since in our model Hertz was best for people renting a car full time for a week, we assume each Hertz user uses the vehicle for one week on average. One-way models probably have to user more cars per user than Zipcar round-trip does because the cars are not all in a central location. For this reason, we will then assume a ratio of 60 users per car in one-way models. Using the pricing models for each car-sharing option, the following results are obtained:

Car-sharing method	Revenue per user per year ($)	Cost per user per year ($)	(Revenue – Cost) per user per year
Round-trip Zipcar	762	126.9	635.1
Round-trip Hertz	214	126.9	87.1
Free-floating Car2go	1641	190.4	1450.6
One-way station Zipcar Boston	675.25	190.4	484.85

For the total profit, we need to multiply (*Revenue – Cost*) per user per year by the number of users. An individual will be incentivized to switch to car-sharing only if car-sharing is the cheapest option, and that will be true only if individual salary exceeds a certain number. And the company will benefit only if it can maximize the number of users as well as minimize the number of rental stations. We are looking for the greatest *number* of people per unit area with enough money to afford car-sharing. The cities with the greatest population densities, and thus the greatest number of people who can afford car-sharing, are Poughkeepsie, NY (density 5983/mi^2); Richmond, VA (3625); Riverside, CA (3085); and Knoxville, TN (1861).

As median per capita income is relatively low and relatively similar for every city, so we assume that each city has the same per capita income distribution. Thus we assume that each city has a proportion k (a very small number) of individuals will have enough money to participate in ride sharing in each city. With this in mind, the total profit of a company will be

Profit=(revenue–cost)*users=(revenue–cost)*k*population density.

For each city, the profit is

	Poughkeepsie, NY	Richmond, VA	Riverside, CA	Knoxville, TN
Round-trip Zipcar	3.80E+06*k	2.30E+06*k	1.96E+06*k	1.18E+06*k
Round-trip Hertz	5.21E+05*k	3.16E+05*k	2.69E+05*k	1.62E+05*k
Free-floating	**8.68E+06*k**	**5.26E+06*k**	**4.47E+06*k**	**2.70E+06*k**
One-way	2.90E+06*k	1.76E+06*k	1.50E+06*k	9.02E+05*k

While these data seem to show that Car2go's free-floating model is much better than any other model, it is important to consider that because of the significantly higher price and lower user to car ratio, the company probably will not have as many users.

The most profitable options for a car-sharing company are the free-floating model and the round-trip Zipcar model. The most economical options for an individual (with enough salary) are individual ownership, joint ownership, free-floating Car2go, and one-way station Zipcar Boston. As a car-sharing company cannot capitalize on the individual or joint ownership market, the best options for them, in terms of individual participation, are still the free-floating model and one-way station model.
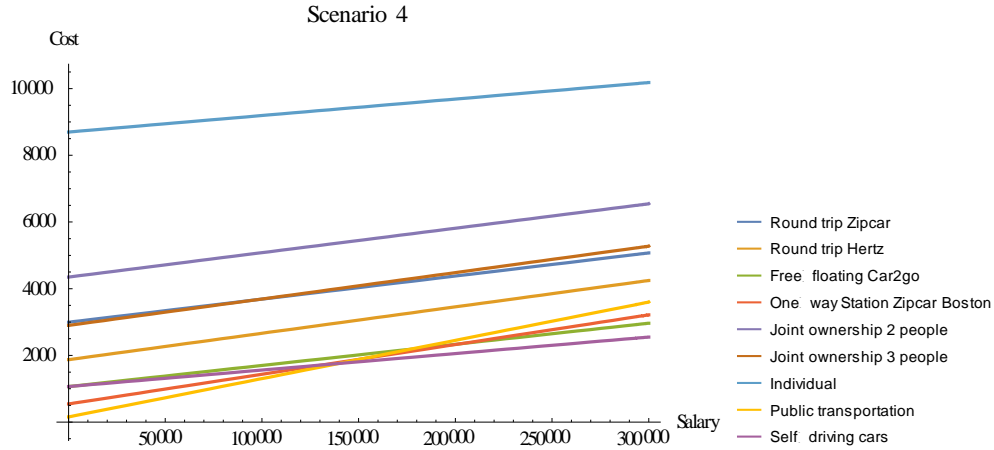
Thus, a car company should invest in developing a free-floating car-sharing system for the cities of Richmond, VA, and Poughkeepsie, NY. Richmond, VA, has more individuals overall that are able to afford car-sharing, but Poughkeepsie, NY, has more individuals within a given area, and thus a company can profit more per station built.

Ranked, the best cities for a company to develop car-sharing in are Poughkeepsie, NY; Richmond VA; Riverside, CA; and Knoxville, TN. The best business models to pursue are the free-floating model and the one-way model. (The one-way model and the round-trip Zipcar model are relatively equal in terms of profit, but we found earlier that an individual is much more likely to use a one-way car.)

D. Limitations
We assumed that the per capita incomes of each city were relatively equal because the discrepancies we found were not very significant. To improve on our model, we would take those differences into account in estimating how many individuals would be able to afford car-sharing. Even better, if we were able to find a per capita income distribution for each of the cities, we could improve on our model even more by having a more definitive number for individuals able to participate, rather than a rough estimate.

When determining profitability of each model, did not incorporate the cost of the jockey for the free-floating Car2go model because we ran out of time. However, given that it was more profitable than the next best option by close to 300%, this additional cost likely would not have made a big difference in the overall profitability of that model.

Part III: Road Map to the Future

A. Restatement of Problem

Alternative energy vehicles and self-driving vehicles have the ability to dramatically alter the car-transportation business. The incredible growth of Uber has demonstrated that by eliminating the hassle of commuting to a car-sharing station without a car, the cost benefits of car-sharing can truly monopolize the market (Los Angeles Times). Self-driving vehicles would allow users to have vehicles delivered directly to their doorstep, replicating the convenience of having a POV. Many people also choose public transportation, biking, or walking over vehicular transport because they want to consume less fuel. For this reason, car-sharing alternative energy vehicles would allow users to be environmentally friendly and take advantage of the convenience of a vehicle.

B. Assumptions

1) Self-driving vehicles would allow all car-sharing businesses to be one-way because the car could drive itself back to any needed location. Stations would not be required because of the self-driving function, so all options would resemble the one-way floating option.
2) Users would stay within a certain radius to avoid exorbitant costs of having the car drive itself back between cities.

C. Analysis

The self-driving feature would eliminate the cost-opportunity of the time taken to reach a car-sharing region. Using the pricing model of the one-way floating model of Car2Go, the self-driving car option comes out as the best option in both Scenario 3 and Scenario 4.

Scenario 4

Not only would it increase the benefit for the user in terms of time saved, but it would also benefit the car-sharing company by decreasing fuel cost and by increasing usage due to the environmentally friendly perk. However, it takes an electric car at minimum 4 hours to completely recharge, and it can only go about 100 miles. Since the average speed we found in Part I was 30.99 mph, this means that an electric car must spend about 60% of its time charging as opposed to being a profitable asset for the company. This lowers the free-floating model's revenue per user, as shown in the chart below. In addition, the cost of charging the car ends up about equaling the cost of fuel due to the recent plummet of fuel prices (Forbes).

Car-sharing method	Revenue per user per year ($)	Cost per user per year ($)	(Revenue – Cost) per user per year
Free-floating Car2go	1641	190.4	1450.6
Self-driving car	875.416	190.4	685

D. Conclusions

This model does not take into consideration the fact that self-driving cars and environmentally-friendly cars are much more expensive than regular sedans. Right now the price of self-driving cars is far too high for a feasible model. However, once self-driving cars become more commonplace, they will also increase usage. People without licenses and people who would otherwise not meet the safety requirements for car-sharing companies would suddenly become a target group for marketing. Over time, self-driving cars and energy-efficient vehicles would become more cost-effective, moving them solidly into the limelight of the car-sharing industry.

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